MAT1505-03/04 15F Quiz 9 Print Name (Last, First) ________________, ______________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. Use the geometric series algebra manipulation trick to find a power series for $f(x) = \dfrac{3}{4+x^2}$. What is its radius of convergence?

2. Find the radius of convergence and the interval of convergence (use interval notation) for the power series $\sum_{n=1}^{\infty} \dfrac{x^n}{3(n+2)2^n}$. Justify your claims.

▶ solution

① $$\frac{3}{4+x^2} = \frac{3}{4(1+\frac{x^2}{4})} = \frac{3}{4}\,\frac{1}{1-\underbrace{(-\frac{x^2}{4})}_{r}} = \frac{3}{4}\overbrace{\sum_{n=0}^{\infty}\left(-\frac{x^2}{4}\right)^n}^{\sum_{n=0}^{\infty} r^n} = \sum_{n=0}^{\infty} \frac{3(-1)^n x^{2n}}{4\,4^n}$$

$$= \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{3x^{2n}}{4^{n+1}}}$$

$|r|<1$: $\left|-\frac{x^2}{4}\right| < 1 \to |x^2| < 4$

$x^2 < 4$

$|x| < \boxed{2 = R}$ radius of convergence

② $$\sum_{n=1}^{\infty} \frac{x^n}{\underbrace{3(n+2)2^n}_{a_n}}:$$ ratio test

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{|x|^{n+1}}{3(n+3)2^{n+1}}}{\frac{|x|^n}{3(n+2)2^n}} = \frac{|x|^{n+1}}{|x|^n}\,\frac{2^n}{2^{n+1}}\left(\frac{n+2}{n+3}\right)$$

$$= \frac{|x|}{2}\left(\frac{n+2}{n+3}\right) \xrightarrow{n\to\infty} \frac{|x|}{2} < 1 \to |x| < \boxed{2 = R}$$ radius of convergence

check $|x| = 2$: $x = \pm 2$ (ratio test inconclusive)

$$\sum_{n=0}^{\infty} \frac{(\pm 2)^n}{2^n 3(n+2)} = \sum_{n=0}^{\infty} \frac{(\pm 1)^n}{3(n+2)}$$

$+1$: $\sum_{n=0}^{\infty} \frac{1}{3(n+2)} \overset{n\to\infty}{\sim} \sum_{n=0}^{\infty} \frac{1}{3n}$ multiple of divergent $p=1$ series so diverges

-1: $\sum_{n=0}^{\infty} \frac{(-1)^n}{3(n+2)}$ $\quad \frac{1}{3(n+2)} \to 0$ decreasing sequence so converges by alternating series test.

so interval of convergence:

$-2 \le x < 2$ or $\boxed{[-2, 2)}$