MAT1505-03/04 15F Quiz 5   Print Name (Last, First) ________________, ______________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. The curve $y = -h\ln\left(1-\dfrac{x}{R}\right)$ over the interval $0 \le x < R$ is rotated around the x-axis, with $h > 0$, $R > 0$.

a) Let $H > 0$ designate the average value of y over this interval. Write down a simplified definite integral for H and then evaluate it by hand showing all your steps, and make sure it agrees with Maple's evaluation.

b) Make a diagram of this curve over this interval and include a horizontal line for this average value. Does the rectangle it makes appear to have the same area as that above the curve? Explain.

c) Let V denote the volume of a solid formed by revolving this curve segment around the x-axis, and indicate a typical vertical cross-section of the integration region needed to evaluate this volume, labeling its endpoints appropriately to justify your limits of integration and indicating how you obtained your integrand. Write down a simplified definite integral for V and then use Maple to evaluate it.

d) Compare this solid with a cylinder of revolution about the x-axis. What radius would the cylinder need to have for it to have the same volume?

▶ solution

① a) $\boxed{H = \frac{1}{R}\int_0^R -h\ln\left(1-\frac{x}{R}\right)dx}$

$$\int \underbrace{\ln\left(1-\tfrac{x}{R}\right)}_{u}\underbrace{dx}_{dv} = \underbrace{x}_{v}\underbrace{\ln\left(1-\tfrac{x}{R}\right)}_{u} - \underbrace{\int -\frac{x\,dx}{R\left(1-\frac{x}{R}\right)}}_{v\,du}$$

$du = \frac{-1}{R}\frac{dx}{1-\frac{x}{R}}$, $v = x$

$$= x\ln\left(1-\tfrac{x}{R}\right) + -\int \frac{x\,\overbrace{(-dx/R)}^{dw}}{\underbrace{1-x/R}_{w}}$$

$dw = -\frac{dx}{R}$

$\frac{x}{R} = 1-w$, $x = R(1-w)$

$$= \int \frac{R(1-w)\,dw}{w} = \int \frac{R}{w} - R\,dw$$

$= R\ln w - Rw$

$= R\ln\left(1-\frac{x}{R}\right) - R\left(1-\frac{x}{R}\right)$

$= (x-R)\ln\left(1-\frac{x}{R}\right) + (R-x)$

$$H = -\frac{h}{R}(x-R)\left(\ln\left(1-\tfrac{x}{R}\right)-1\right)\Big|_0^R = \left[0(\overset{0}{\ln 1}-1) - (-R)(\overset{0}{\ln 1}-1)\right]\left(-\tfrac{h}{R}\right)$$

$= \boxed{h}$

b) $y = h\Big(\underbrace{-\ln\big(\overbrace{1-\tfrac{x}{R}}^{\text{proper fraction}}\big)}_{\text{negative}}\Big) \to \infty$ as $x \to R^-$ (from the left)

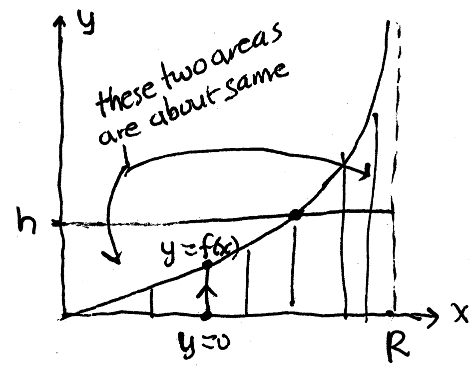
positive.

decreases as $x \to R$ so y is an increasing function of x

$x = 0$: $y = -h\ln 1 = 0$

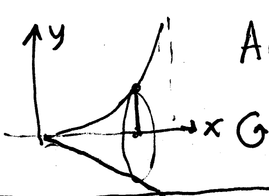
easier: set $h = 1 = R$ to get plot with y & x measured in multiples of h & R.

> plot(-ln(1-x), x = 0..1)



Let $y(x) = -h\ln\left(1-\frac{x}{R}\right)$

c)



$A(x) = \pi f(x)^2$

$$\boxed{V = \int_0^R \pi h^2\left(\ln\left(1-\tfrac{x}{R}\right)\right)^2 dx}$$

$\underset{\text{Maple}}{=} \boxed{2\pi h^2 R}$

d) $V_{cyl} = \pi r^2 R = 2\pi h^2 R$

$\hookrightarrow 2h^2 = r^2 \to \boxed{r = \sqrt{2}\,h}$