MAT1505-03/04 15f Quiz 2 Print Name (Last, First) __________________, ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. a) Evaluate $\int x^2 \sqrt{x^3+1}\, dx$.

b) Transform this definite integral $\int_0^2 x^2 \sqrt{x^3+1}\, dx$ to a much simpler one in an obvious choice of new variable, then evaluate it.

c) Check your new integral value by evaluating the original definite integral using Maple.

▶ **solution**

a) $\int x^2 \sqrt{x^3+1}\, dx = \int u^{1/2} \left(\frac{du}{3}\right) = \frac{1}{3}\int u^{1/2} du = \frac{1}{3}\frac{u^{3/2}}{3/2} + C = \frac{2}{9}u^{3/2} + C$

$u = x^3+1$

$du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

$= \boxed{\frac{2}{9}(x^3+1)^{3/2} + C}$

b) $\int_0^2 x^2\sqrt{x^3+1}\, dx = \frac{1}{3}\int_{x=0}^{x=2} u^{1/2} du = \boxed{\frac{1}{3}\int_1^9 u^{1/2} du}$

$x=2 \rightarrow u = 2^3+1 = 9$

$x=0 \rightarrow u = 0+1 = 1$

$= \frac{1}{3}\frac{u^{3/2}}{3/2}\Big|_1^9 = \frac{2}{9}u^{3/2}\Big|_1^9$

$= \frac{2}{9}(9^{3/2} - 1^{3/2}) = \frac{2}{9}(3^3 - 1)$

$= \frac{2 \cdot 26}{9} = \boxed{\frac{52}{9}}$ ≈ 5.778

optional here

c) $> \int_0^2 x^2\sqrt{x^3+1}\, dx$

$\underset{\text{Maple}}{=} \frac{52}{9}$ ✓ agrees