

Mat1505-Jantzen 15F. Test 2 comments.

**Problem 1.**

See the Chapter 8 Review problem 9. With the derivative of the definite integral evaluated by the fundamental theorem of calculus, you get the integrand as a function of  $x$ , then squaring and adding 1 you get a perfect square (namely  $\sqrt{x}$ ) with no extra algebra of interpreting a binomial theorem perfect square:

9. Find the length of the curve

$$y = \int_1^x \sqrt{\sqrt{t} - 1} dt \quad 1 \leq x \leq 16$$

I thought this would be the easiest way to make a simple length of curve problem. Many of you ignored my instruction to find the length of the graph of the function  $f$  and instead attacked the function whose area this represents, leading to a horrible integral that could only be done numerically.

This latter wrong value was about  $1/4$  longer than the approximate secant line length (joining the endpoints) since the ratio of the numerical value 4.33 compared to the secant value 3.41 is about 1.27, and if you extend the secant line by  $1/4$  and compare it to the curve, you see there is no way by straightening out the curve you will get it to stretch out that far.

I suggested looking at problems 1-9, 21-23 of the review problems over the weekend before the test, and asked on Monday and Tuesday if anyone had any questions about them. No one did. I intended this to be a really simple problem that would avoid the algebra manipulations needed for most perfect square such problems.

**Problems 2, 3.**

For improper integrals on a finite closed interval, the minimum we need to be able to do is identify where a vertical asymptote occurs and state the limit needed to assure that the result really does give the integral we want if finite. On a semi-infinite integral, we need to evaluate the contribution to the antiderivative evaluation at the infinite limit of integration to make sure it is finite, so we need at least to state that limit and make sure that its value is obvious. Any indeterminate form limits need to be evaluated, typically by l'Hopital's rule.

In Problem 2, there is division by zero in the integrand at the left limit  $y = 2$ , so we need to take a limit as  $y$  approaches 2 from the right, i.e.,  $2+$ . Some people put a limit as  $y$  approached  $y = 4$  just to use a limit somewhere. The antiderivative is continuous at  $y = 2$  so it is enough to evaluate it there, but the limit should have been stated.

In Problem 3, on the positive  $x$  axis only (some people wrote down integrals with a negative infinity lower limit, but the lower limit on the integration range is 0), it states, use technology for the antiderivative, yet many students wasted time trying an integration by parts, some of which got derailed there. I stated on line, in an email and verbally that for 1d probability distributions, you should know how to calculate the expected (or mean or average) value (review problem 21 asked you to evaluate the mean value of a distribution), defined to be simply inserting a factor of the random variable into the integral for total probability and evaluating the new integral. For the peak value, it is easy to differentiate this product by hand and immediately find by setting it equal to zero that the peak occurs

at  $x = a$ . If you set  $a = 1$ , like in the maple HW, you can plot this distribution for say  $x = 0 \dots 6$ , and you see with gridlines that indeed the peak is at  $x = 1$ . You can also see there is more area under the curve to the right of the peak than to the left, so the mean value will be pushed to the right of the peak value. For  $x = 4$ , you see only a small tail left, so the probability up to  $x = 4a$  in the original distribution should be very small, as indeed you find. If you got lost, it would have taken tens of seconds to simply evaluate all the integrals in maple to at least get the correct answer for partial credit.

Problem 4 was a quickie, just to see if you really understood Simpson's rule for approximating an integral. 2 divisions, orally confirmed to be 2 subintervals, the minimum value to apply Simpson's rule, made it the shortest application I could think of. Since this method is exact for quadratic functions (the quadratic approximation to a quadratic function is itself, which is how Simpson evaluates the integral), a short hand calculation by hand (or by technology if you state it as required by the instructions) you see the exact result. By evaluating the Simpson approximation exactly (no decimals) you reproduce the same result. Easy.

I had thought this would be a straightforward test, but a minimal of thinking was required beyond doing routine homework for one section at a time. Calc courses are only valuable to the extent to which they open up your mind to understand how functions behave using their tools, and how to use mathematics to implement the ideas of calculus in applied problems. Mathematical thinking is the goal, widening your understanding of functional relationships and how they might be reflected in real world problems. Repeating rote problems mechanically serves no one.