# MAT1505-03/04 15F Test 1 Answers

① a) $\overset{f(x)=}{2x^2-6x+5} = \overset{g(x)=}{-x^2+6x-4}$

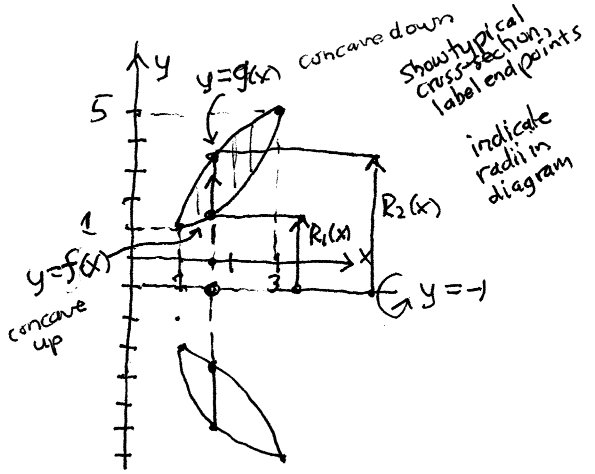
$3x^2-12x+9=0$

$x^2-4x+3=0$

$(x-1)(x-3)=0 \rightarrow x=1,3$

$x=1: \quad y=2-6+5=1$

$x=3: \quad y=18-18+5=5$




$R_2(x) = g(x)-(-1) = -x^2+6x-4+1$
$\qquad = -x^2+6x-3$

$R_1(x) = f(x)-(-1) = 2x^2-6x+5+1$
$\qquad = 2x^2-6x+6$

$$V = \int_1^3 \pi \left(R_2(x)^2 - R_1(x)^2\right) dx$$

$$= \boxed{\pi \int_1^3 -(2x^2-6x+6)^2 + (-x^2+6x-3)^2\, dx}$$

$$\underset{\text{Maple}}{=} \pi \int_1^3 -3x^4+12x^3-18x^2+36x-27\, dx$$

$$\underset{\text{Maple}}{=} \boxed{\frac{144\pi}{5}}$$

② a) $T_{avg} = \dfrac{1}{12}\displaystyle\int_0^{12} 50+4t\sin\frac{\pi t}{12}\, dt$

$$= \frac{1}{12}\int_0^{12} 50\,dt + \frac{4}{12}\int_0^{12} t\sin\frac{\pi t}{12}\, dt$$

$$= \frac{50}{12}t\Big|_0^{12} + \frac{1}{3}\int_0^{12} \underbrace{t}_{u}\ \underbrace{\sin\frac{\pi t}{12}\, dt}_{dv}$$

$u=t \qquad dv = \sin \pi t/12$

$du = dt \qquad v = \int dv = -\dfrac{\cos \pi t/12}{\pi/12}$

② continued

$$T_{avg} = \frac{50}{12}(12-0) + \frac{1}{3}\left[ t\left(-\frac{12}{\pi}\cos\frac{\pi t}{12}\right)\Big|_0^{12} \underbrace{- \int_0^{12} -\frac{12}{\pi}\cos\frac{\pi t}{12}\, dt}_{+\frac{12}{\pi}\frac{\sin\frac{\pi t}{12}}{\pi/12}\Big|_0^{12}}\right.$$

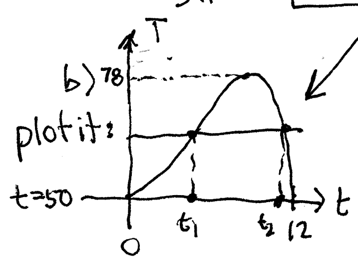
$$= 50 + \frac{1}{3}\left[-\frac{12}{\pi}t\cos\frac{\pi t}{12} + \left(\frac{12}{\pi}\right)^2 \sin\frac{\pi t}{12}\right]_0^{12}$$

$$= 50 + \frac{1}{3}\left[-\frac{12}{\pi}(12)\underbrace{\cos\pi}_{-1} - 0 + \left(\frac{12}{\pi}\right)^2(0-0)\right]$$

$$= 50 + \frac{12^2}{3\pi} = \boxed{50+\frac{48}{\pi}} \approx \boxed{65.3\ ^\circ F}$$

makes sense in plot!

b) plot it:




$50 + 4t\sin\frac{\pi t}{12} = 50 + \frac{48}{\pi}$

two solutions:

$\boxed{\begin{array}{l} t_1 \approx 4.256 \\ t_2 \approx 10.591 \end{array}}$ $\xrightarrow{c)}$ 1:15pm
$\longrightarrow$ 7:35pm

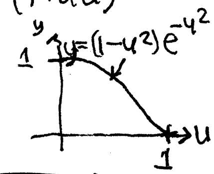
need 4 digits here to be sure of nearest minute!

③ $V_{avg} = \dfrac{1}{R}\displaystyle\int_0^R \frac{P}{4\eta l}(R^2-r^2)e^{-\frac{r^2}{R^2}}\, dr$

$u = \frac{r}{R} \quad du = \frac{dr}{R} \quad dr = R\,du \qquad r=0 \rightarrow u=0$
$\qquad\qquad\qquad\qquad r = Ru \qquad r=R \rightarrow u=1$

$$= \frac{1}{R}\left(\frac{P}{4\eta l}\right)\int_0^1 \underbrace{(R^2-(Ru)^2)}_{R^2(1-u^2)} e^{-u^2}(R\,du)$$

$$= \underbrace{\frac{R^3}{R}\left(\frac{P}{4\eta l}\right)}\ \underbrace{\int_0^1 (1-u^2)e^{-u^2}\, du}$$




$\boxed{k = \dfrac{R^2P}{4\eta l}}$ $\qquad \boxed{\begin{array}{c} U = \frac{1}{2e} + \frac{\sqrt{\pi}}{4}\operatorname{erf}(1) \\ \approx 0.55736 \end{array}}$

b) $= V_{max}\ ! = v(0)$ $\qquad = V_{avg}/V_{max}$

④ $\displaystyle\lim_{h\to 0} \frac{1}{h}\int_2^{2+h} \sqrt{1+x^3}\, dx$, let $F'(x) = \sqrt{1+x^3}$

$$= \lim_{h\to 0} \frac{1}{h}\left(F(2+h) - F(2)\right) = F'(2)$$

$$= \sqrt{1+2^3} = \boxed{3}$$