MAT1505-03/04 15F Test 1 Print Name (Last, First) _____ , _____|____
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

Optional parts are only for students who have completed the remaining questions to their satisfaction.

1. $y = 2x^2 - 6x + 5, y = -x^2 + 6x - 4.$
Write down a definite integral for the volume of the solid of revolution which results from revolving the region R around the axis $y = -1$. Support your choice of integrand and limits of integration with a fully labeled diagram shading the region of integration and including an appropriately labeled typical cross-section and identifying the ingredients in your integrand in the diagram. Then evaluate your integral with technology.

2. The temperature in degrees Fahrenheit for a 12 hour period from 9am to 9pm is $T = 50 + 4t \sin\left(\frac{\pi t}{12}\right)$, with t the time in hours starting at 9am. a) Find the average temperature during this period exactly and to the nearest tenth of a degree, showing all steps in the hand integration process.
b) Find the times t when the temperature equals the average temperature.
c) **Optional.** Find the clock times (as in 10:31am) to the nearest minute when the temperature equals the average temperature.

3. Given the velocity profile $v(r) = \frac{P}{4\eta\ell}\left(R^2 - r^2\right) e^{-\frac{r^2}{R^2}}$ for $0 \le r \le R$, write down a definite integral for its average value v_{avg} and rewrite this integral in terms of the dimensionless variable $u = \frac{r}{R}$ and simplify to the form $k\,U$ where U is a dimensionless integral involving no parameters. What is the exact and numerical value of U? [Use Maple to evaluate this final integral.]

b) **Optional:** What is the value of the ratio $\frac{v_{avg}}{v_{\max}}$ (exact and to 4 decimal places), where $v_{\max}$ is the maximum value of the velocity on this interval? Use this result to make a simple sketch of the velocity profile and its average value (as a horizontal line).

4. What does the Fundamental theorem of Calculus say about evaluating this limit: $\lim_{h \to 0} \frac{1}{h} \int_2^{2+h} \sqrt{1 + x^3}\, dx$.

Explain how you evaluated it.
[Hint: let F be an antiderivative of this integrand.]

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: