MAT2500-03/04 14S Quiz 9 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. a) Set up an interated double integral in polar coordinates for the volume between the planes $z=0$ and $y+z=3$ inside the cylinder $x^2+y^2=4$. Evaluate it in Maple.
b) Now evaluate it step by step by hand.

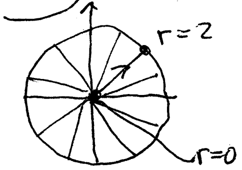
2. Consider the integral $\int_0^2\int_0^{\sqrt{2y-y^2}} x\,dx\,dy$.
a) Draw a diagram shading the region of integration, properly labeling the bullet endpoints of a typical cross-section line segment.
b) Now redraw the diagram appropriate for polar coordinates, and write down the corresponding iterated double integral.
c) Evaluate the two integrals with Maple to make sure they agree.

▶ solution

① $z=0$, $y+z=3 \to z=3-y>0$ for y in circle of radius 2: Volume under its graph

$\underbrace{x^2+y^2}_{r^2}=4 \to r=2$

$$V=\boxed{\int_0^{2\pi}\int_0^2 3-(r\sin\theta)\,r\,dr\,d\theta} \overset{\text{Maple}}{=} \boxed{12\pi}$$



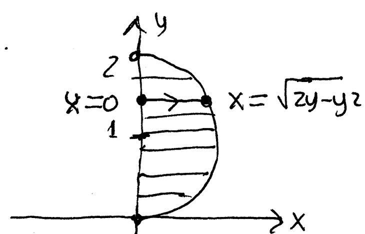
$\theta=0..2\pi$

(b)

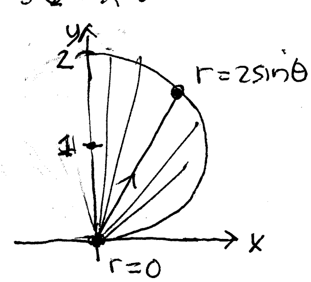
$$\int_0^2 3r-r^2\sin\theta\,dr = \frac{3r^2}{2}-\frac{r^3}{3}\sin\theta\Big|_0^2$$
$$=3\cdot2-\frac{8}{3}\sin\theta = 6-\frac{8}{3}\sin\theta$$
$$=\int_0^{2\pi} 6-\frac{8}{3}\sin\theta\,d\theta = 6\theta+\frac{8}{3}\cos\theta\Big|_0^{2\pi} = 6(2\pi)+\frac{8}{3}(1-1)$$
$$=\boxed{12\pi}$$

② $\int_{y=0}^{y=2}\int_{x=0}^{x=\sqrt{2y-y^2}} x\,dx\,dy$

a) $x^2=2y-y^2$, $x^2+y^2=2y$ (clearly a circle)
$=y(2-y)$ $\quad r^2=2r\sin\theta \to r=2\sin\theta$
$=0$
at $y=0, 2$.



b)



$\theta=0..\pi/2$

$$\boxed{\int_0^{\frac{\pi}{2}}\int_0^{2\sin\theta} \underbrace{(r\cos\theta)\,r}_{r^2\cos\theta\,!}\,dr\,d\theta} \overset{(c)}{\underset{\text{Maple}}{=}} \boxed{\frac{2}{3}} \underset{\text{Maple}}{=} \int_0^2\int_0^{\sqrt{2y-y^2}} x\,dx\,dy$$