MAT2500-03/04 14S Final Exam Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL signs and arrows when appropriate. Always SIMPLIFY expressions. LABEL parts of problem. BOX final short answers. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Use technology to check any integrals you set up.

1. Given the point $(x, y, z) = (12, 9, 8)$, find the new coordinates, in each case stating the angles both in radians (exactly, using inverse trig functions) and in degrees (1 decimal place accuracy) and use proper identifying symbols for all coordinates: a) cylindrical coordinates. b) spherical coordinates. Support your work with two diagrams, one of the x-y plane and one of the r-z half plane, each including a reference triangle locating the point with respect to the axes with all three sides labeled by their lengths and both axes labeled by their coordinate labels and showing the relevant angles measured from their appropriate axes using an arc with an arrow indicating the increasing direction of that angle. Show clearly how you obtain values of your coordinates from these diagrams. Do the angles look right in these diagrams?

2. a) Express $\iint_R f\, dA = \int_0^4 \int_0^{\frac{3}{4}x} f\, dy\, dx + \int_4^5 \int_0^{\sqrt{25-x^2}} f\, dy\, dx$ as a single double integral using the reverse order of integration, supporting your work first with a completely labeled diagram of the region R of integration, indicating typical bullet endpoint cross-section line segments describing each of these two iterated integrals (arrow in the middle indicating the direction of the integrating variable, endpoint values of that variable stated), and then a similar second diagram appropriate for the reversed order of integration.
b) Now redo the diagram appropriate for polar coordinates and re-express the integral in those coordinates.
c) Evaluate all three expressions for $f = 1$ both exactly (no decimals in expression) and numerically to 4 decimal places to make sure they agree and give the area of R. Use technology as you see appropriate.
d) Verify Green's Theorem $\int_C \vec{F} \cdot d\vec{r} = \iint_R \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dA$ for the region R and the vector field $\vec{F} = \langle x - y, x + y \rangle$ around the counterclockwise bounding curve $C = C_1 + C_2 + C_3$ of R.

3. Consider the solid of revolution region R [half cutaway view on reverse page] whose outer wall surface is the sphere $x^2 + y^2 + z^2 = 17$, whose ceiling surface is the upper half cone $x^2 + y^2 = 16\,z^2$, $z \geq 0$ and whose floor surface is the plane $z = 0$.
a) Make a labeled diagram of the r-z half plane appropriate to do an integral in spherical coordinates showing a typical radial cross-section and a labeled arc showing the starting and stopping values of the angle ϕ measured from the upper z-axis.
b) What is the height of this solid region? Indicate this value on the z-axis of your diagram. At what cylindrical radius r does the upper rim of this solid region occur? Indicate this value on the r-axis of your diagram.
c) Evaluate the two integrals step by step exactly: $V = \iiint_R 1\, dV$, $VZ = \iiint_R z\, dV$, and give their exact and numerical values and of their centroid height ratio $\bar{z}$ to 4 decimal places. (Note that this final exact result should be a simple rational number.) Mark this value on the z-axis of your diagram. Does it look right? Explain.

4. $\vec{\boldsymbol{F}} = \langle x, y, z \rangle$
a) Show that $\vec{\boldsymbol{F}}$ satisfies the curlfree condition that it admit a potential function, i.e., is a conservative vector field.

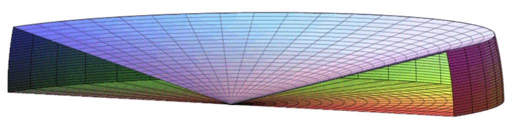
b) Find a potential function f for it.

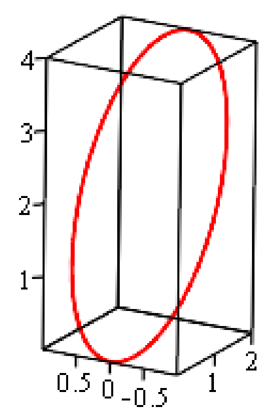
c) Use the potential to evaluate the line integral $\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ over any curve from $P(0, 0, 0)$ to $Q(2, 0, 4)$.

d) Check your result by doing the line integral on a straight line segment.

e) Consider the closed curve $\vec{r}(t) = \langle 2\cos(t)\cos(t), 2\cos(t)\sin(t), 4\cos^2(t)\rangle$ for $t = 0..\pi$ shown below, which is confined to the surface of revolution $z = r^2$ in cylindrical coordinates. Confirm this statement. Then show that the parametrized curve C_2: $t = \frac{\pi}{2} .. \pi$ also runs from $P(0, 0, 0)$ to $Q(2, 0, 4)$.

f) Evaluate $\int_{C_2} \vec{F} \cdot d\vec{r}$ directly from this parametrized curve. Comment on the relationship between c), d) and this last line integral.





solution (on-line)

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in with your answer sheets as a cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: