MAT2500-03/04 14S Takehome Test 3 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Explain in as many words as possible everything you are doing! For each hand integration step, state the antiderivative formula used before substituting limits into it: $\int_a^b f(x)\ \mathrm{d}x = F(x)|_{x=a}^{x=b} = F(b) - F(a)$.

1. **Double Integral.** $\int_2^4 \int_{-\sqrt{4y-y^2}}^0 x\ \mathrm{d}x\ \mathrm{d}y$.

a) Do this integral by hand.
b) Make a completely labeled (shaded by typical cross-sections) diagram of the region of integration for this integral, with a typical correctly labeled cross-section line segment (bullet endpoints) representing the current iteration of the integral.
c) Redo this diagram appropriate for the reversed order of integration and evaluate it exactly using technology.
d) Redo this diagram appropriate for a polar coordinate iteration of the integral, indicating the appropriate range for the polar angle on the diagram.
e) Evaluate the new integral by hand, using simple u-substitutions.

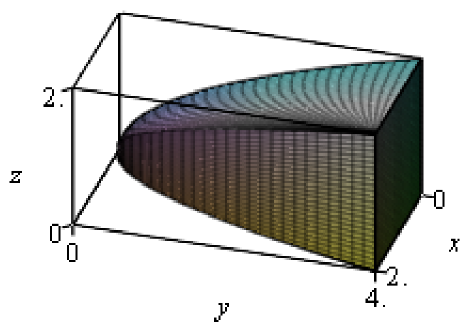
2. **Space Integral.** Consider the solid region R in the first octant enclosed by the surfaces $z=\sqrt{y}$, $y=x^2$, $y=4$ and the x-y plane.
See the figure on page 2 (left).
a) Set up an iterated triple integral $V=\iiint 1\ dV$ for the volume of this region in the order $dV=dx\ dz\ dy$ and evaluate it step by step by hand exactly and give its numerical value to 4 decimal places. Support your limits of integration with a diagram for the outer double integral with completely labeled line segment cross-sections and equally spaced such cross-sections for the shading, and a similar diagram for the innermost integral.
b) Repeat for the order $dV=dz\ dy\ dx$.
c) Check your two integrals exactly using technology, reporting Maple's results. They should agree with each other and your own evaluations. Do they?

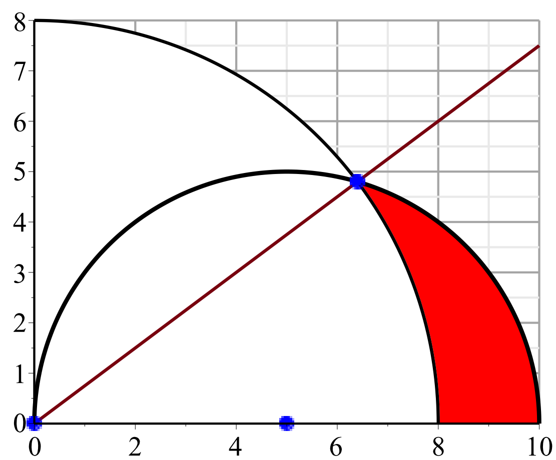
3. **Weird Torus.** Consider the solid of revolution E shown in the figure of the r-z half plane (page 2 right) obtained by rotating the darker shaded region about the (vertical) z-axis. Shown are a circle of radius 8 centered about the origin (inside E), a circle of radius 5 centered at a point 5 units along the horizontal r-axis (outside E), and the half line from the origin through their point of intersection. By symmetry the centroid of this solid will lie on the z-axis, at height $\bar{z}$.
a) Write equations in standard form for the two circles expressed in terms of their radii and centers in the cylindrical coordinates (r, z).
b) Find the cylindrical coordinates (R, Z) of their intersection, and write an equation in cylindrical coordinates for the half line through the intersection point.
c) Now re-express the equations for those two circles in spherical coordinates (ρ, ϕ), solving each for the radial coordinate.
d) Evaluate the spherical coordinates of the intersection point.
e) Set up a spherical coordinate integral for the volume V and xy-moment $M_{xy}=\iiint z\ dV$ of E, and support your limits of integration for the inner double integral in the r-z half plane with a completely labeled bullet endpoint typical line segment cross-section accompanied by shading of the region by equally spaced cross-sections.
f) Evaluate these integrals step by step by hand exactly and then numerically approximate them and their exact ratio $\bar{z}$ to 2 decimal places.
g) Now repeat 3) for a radial first integration in cylindrical coordinates.
h) Evaluate these integrals exactly using Maple and evaluate their numerical ratio $\bar{z}$ to 2 decimal places. [Verify that Maple gives the same exact formulas for both coordinate systems.]
i) Mark your centroid point on one of your diagrams, identifying it. Does this value of the centroid height seem reasonable? Explain its value in relation to half the height of this solid.

4. **Cartesian to spherical coordinate integral conversion.**

a) Use spherical coordinates to represent $\int_0^3 \int_{-\sqrt{9-x^2}}^{0} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} x^2+y^2+z^2 \, dz \, dy \, dx$ as a triple integral, supporting your limits of integration with labeled diagrams as above.
b) Evaluate this new integral exactly with technology and compare its value with the original triple integral in Cartesian coordinates evaluated exactly and numerically with technology. Do they agree?
c) Evaluate the spherical coordinate integral exactly by hand step by step. Does it agree with your previous results?

Note that the origin is in the back left corner of the left diagram.





▶ solution (on-line)

No collaboration. You may only talk to bob. See test rules on-line. Read short rules above. Print out and attach any Maple supporting work you do, hand annotating if necessary with problem number and part etc, whatever is necessary for clarification.

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in stapled to your answer sheets as the cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants. "

Signature: Date: