

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). You are encouraged to use technology to check all of your hand results.

1. $f(x, y, z) = 2xy^2z^2 - y^2 - z^3 + 1$; $P(1, -1, 1)$

- Write an equation for the level surface through P .
- Find a unit vector in the direction in which this function increases most rapidly at P . What is its rate of change in this direction?
- Evaluate the directional derivative of f at P in the direction of the origin. Is f increasing or decreasing in this direction? Why?
- Write the simplest equation for the tangent plane to the level surface of f at P .
- Write parametrized equations for the normal line at P .
- Use the linear approximation at P to approximate $f(1.02, -0.98, 1.03)$.

2. The two legs of a right triangle are measured as 8cm and 15cm with a possible error in measurement of at most 0.2cm in each. Use differentials to estimate the maximum error in the calculated value of the length of the hypotenuse. What is the percentage error?

3. $f(x, y) = (x^2 + xy)e^{-y}$.

- Show that $(0, 0)$ and $(-1, 2)$ are the two critical points of f .
- Use the second derivative test to classify these critical points as local minima, local maxima or saddle points. Use words to explain your reasoning.

4. Use the max-min approach step by step to find three positive numbers whose sum is 12 and the sum of whose squares is as small as possible. What is that minimum value? State clearly the domain of the function of 2 variables over which you are looking for your critical point and make a sketch of it.

Optional. After a moment's thought, what geometrical problem involving points and planes did you just solve? Explain.

► solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: _____

Date: _____