MAT2705-01/02 14F Quiz 6 Print Name (Last, First) ______________________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **[You should use technology for row reductions and determinants. Report results you find in this way: value of det, reduced matrix.]**

1. a) Find the general solution of the following linear system, identifying your coefficient matrix A, the rhs vector $\vec{b}$, the augmented matrix, the rref matrix and give the solution in scalar form: $x_1 = \ldots,\ x_2 = \ldots$ *etc.*.

$$2x_1 + 7x_2 - 10x_3 - 19x_4 = 13$$
$$x_1 + 3x_2 - 4x_3 - 8x_4 = 6$$
$$x_1 + 2x_3 + x_4 = 3$$

b) Write the solution in the vector form $\vec{x} = \ldots$ and then express it as an explicit linear combination of a set of vectors (all but one of which have the free parameters as coefficients).

c) Identify a basis of the solution space of the related homogeneous system corresponding to setting $\vec{b} = \vec{0}$. [Write your basis using curly brace set notation, using either angle bracket vector notation or column matrix notation.]

d) Consider the set of vectors $\{\vec{v}_1, \ldots, \vec{v}_4\}$ which form the columns of A. What are the independent relationships among this vectors? (Write each such relationship in the form: a linear combination of them equals the zero vector.)

e) What subset of these vectors does our solution algorithm show to be linearly independent automatically? Express $\vec{b}$ as a unique linear combination of these latter vectors. [The particular solution!]

2. Which of the following sets of vectors are linearly independent? Justify your claim. Report the results of any calculation you use in doing so.

a) $\{\langle 2,5,-1\rangle, \langle 0,-5,4\rangle, \langle -1,2,5\rangle\}$,

b) $\{\langle 0,-2,-1\rangle, \langle -1,1,-1\rangle, \langle 2,-4,1\rangle\}$,

c) $\{\langle 0,1,2,3\rangle, \langle 3,4,4,4\rangle, \langle 1,-1,-2,0\rangle\}$.

▶ solution

① a) $A = \begin{bmatrix} 2 & 7 & -10 & -19 \\ 1 & 3 & -4 & -8 \\ 1 & 0 & 2 & 1 \end{bmatrix}, \vec{b} = \begin{bmatrix} 13 \\ 6 \\ 3 \end{bmatrix}$

$\langle A|\vec{b}\rangle = \begin{bmatrix} 2 & 7 & -10 & -19 & 13 \\ 1 & 3 & -4 & -8 & 6 \\ 1 & 0 & 2 & 1 & 3 \end{bmatrix} \xrightarrow{\text{rref}}$

$\begin{array}{c} \begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & -2 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ L & L & F & F \end{array} \end{array}$ $\begin{array}{l} \to x_1 + 2x_3 + x_4 = 3 \\ \to x_2 - 2x_3 - 3x_4 = 1 \end{array}$

$\to x_3 = t_1,\ x_4 = t_2$

$$\boxed{\begin{array}{l} x_1 = 3 - 2t_1 - t_2 \\ x_2 = 1 + 2t_1 + 3t_2 \\ x_3 = t_1 \\ x_4 = t_2 \end{array}}$$

b) $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 - 2t_1 - t_2 \\ 1 + 2t_1 + 3t_2 \\ t_1 \\ t_2 \end{bmatrix}$

$= t_1 \begin{bmatrix} -2 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -1 \\ 3 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

c) soln space basis: $\left\{ \begin{bmatrix} -2 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$

or $\boxed{\{\langle -2,2,1,0\rangle, \langle -1,3,0,1\rangle\}}$

d) basis provides coefficients of linear relationships:

$$\boxed{-2\vec{v}_1 + 2\vec{v}_2 + \vec{v}_3 = 0,\quad -\vec{v}_1 + 3\vec{v}_2 + \vec{v}_4 = 0}$$

e) $\boxed{\{\vec{v}_1, \vec{v}_2\}}$ leading columns are lin. ind., clearly $\vec{v}_3$ and $\vec{v}_4$ can be expressed in terms of them. (and they alone would reduce to matrix with no free columns, no "nontrivial" solutions)

$\boxed{\vec{b} = 3\vec{v}_1 + 1\vec{v}_2}$ (set $t_1 = t_2 = 0$!)

[check: $3\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 6+7 \\ 3+3 \\ 3+0 \end{bmatrix} = \begin{bmatrix} 13 \\ 6 \\ 3 \end{bmatrix}$ ✓ not required.]

② a) $\begin{vmatrix} 2 & 0 & -1 \\ 5 & -5 & 2 \\ -1 & 4 & 5 \end{vmatrix} = -81 \neq 0 \to \boxed{\text{lin. ind.}}$ (or rref(...) $= I$)

b) $\begin{vmatrix} 0 & -1 & 2 \\ -2 & 1 & -4 \\ -1 & -1 & 1 \end{vmatrix} = 0 \to \boxed{\text{lin dep}}$ or rref(...) $= \begin{array}{c} \begin{array}{ccc} L & L & F \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$

1 ind. linear relationship.

c) $A = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 4 & -1 \\ 2 & 4 & -2 \\ 3 & 4 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ ∴ $A\vec{x} = \vec{0}$ has no nonzero solns so $\boxed{\text{lin. ind.}}$ (no linear relationships)