MAT2705-01/02 14F Quiz 5 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **[You should use technology for row reductions and determinants. Report results you find in this way: value of det, reduced matrix.]**

1. a) $\begin{vmatrix} -2 & 0 & 3 \\ 1 & -1 & -2 \\ 4 & 2 & -1 \end{vmatrix}$. Show the row reduction steps to evaluate the determinant of this matrix. Identify each elementary row operation.

b) Evaluate the determinant with technology. What is the result? Does it agree with part a)?

2. $2\,x_1 - 2\,x_2 = 6,\quad -3\,x_1 + x_2 = -7$ a) Write this linear system in the matrix form $\mathbf{A}\,\vec{x} = \vec{b}$.

b) Write down the inverse coefficient matrix using technology or your memory if good enough, but then verify that its product with **A** is the identity matrix. Show the matrix multiplication steps by hand (sums of products before simplifying) to prove that you can actually multiply simple matrices.

c) Now solve this matrix equation for the column matrix $\vec{x}$ using the inverse matrix, and then write out the individual scalar solutions of the original system for each individual variable.

d) Check by backsubstitution into the original two equations that your solution is actually a solution.

▶ solution

① a) $\begin{vmatrix} -2 & 0 & 3 \\ 1 & -1 & -2 \\ 4 & 2 & -1 \end{vmatrix} \overset{R_1 \leftrightarrow R_2}{=} - \begin{vmatrix} 1 & -1 & -2 \\ -2 & 0 & 3 \\ 4 & 2 & -1 \end{vmatrix} \overset{\substack{R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 - 4R_1}}{=} - \begin{vmatrix} 1 & -1 & -2 \\ 0 & -2 & -1 \\ 0 & 6 & 7 \end{vmatrix} \overset{R_3 \to R_3 + 3R_2}{=} - \begin{vmatrix} 1 & -1 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & 4 \end{vmatrix} = -(1)(-2)(4) = \boxed{8}$

b) $= 8$ via Maple ✓

② a) $2x_1 - 2x_2 = 6$
$-3x_1 + x_2 = -7$

$\underbrace{\begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 6 \\ -7 \end{bmatrix}}_{\vec{b}}$

b) $A^{-1} = \dfrac{\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}}{1(2)-3(2)} = -\frac{1}{4}\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} -1/4 & -1/2 \\ -3/4 & -1/2 \end{bmatrix}}$

$A^{-1}A = -\frac{1}{4}\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} 1(2)+2(-3) & 1(-2)+2(+1) \\ 3(2)+2(-3) & 3(-2)+2(+1) \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

c) $A^{-1}[A\vec{x} = \vec{b}] \to \underbrace{A^{-1}A}_{I}\vec{x} = A^{-1}\vec{b}$, $I\vec{x} = \vec{x}$ $\Big\}\ \vec{x} = A^{-1}\vec{b}$: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ -7 \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} 6-14 \\ 18-14 \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -8 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\therefore\ \boxed{x_1 = 2,\ x_2 = -1}$

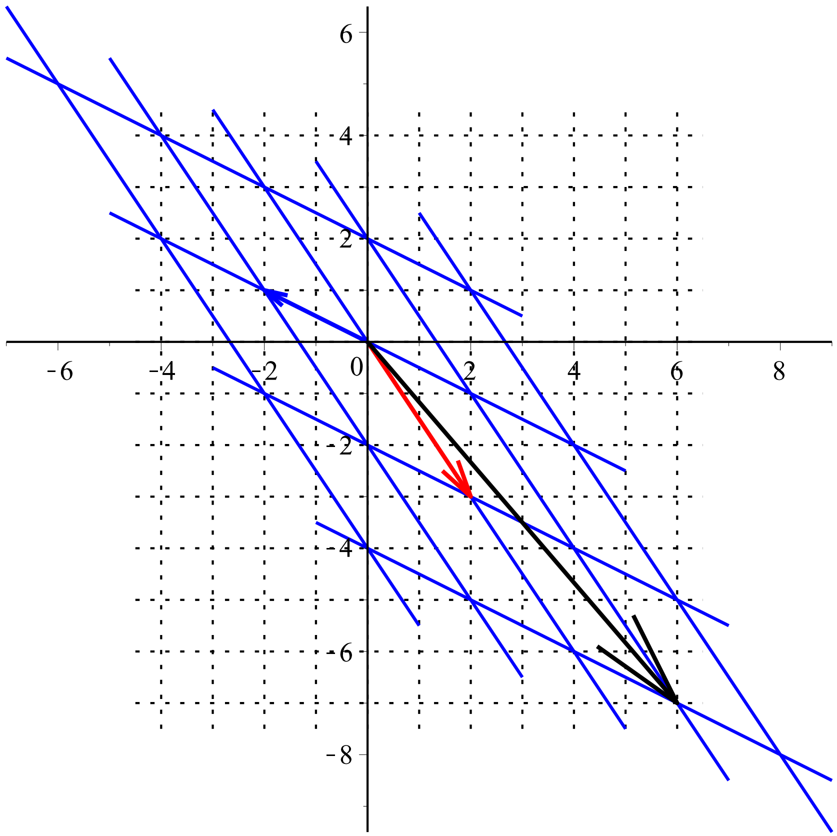
d) $2(2) - 2(-1) = 6?$
$4+2 = 6$
$6 = 6$ ✓

$-3(2) + (-1) = -7?$
$-6 - 1 = -7$
$-7 = -7$ ✓

Note this means:

$2\begin{bmatrix} 2 \\ -3 \end{bmatrix} - \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -7 \end{bmatrix}$

so $(2,-1)$ are the coordinates (components) of $\langle 6,-7\rangle$ with respect to $\{\langle 2,-3\rangle, \langle -2,1\rangle\}$.



Here is the new grid associated with the two columns of the coefficient matrix
and the interpretation of what we just did graphically:
black vector = 2 red vector -- blue vector