MAT2705-01/02 14F Quiz 4   Print Name (Last, First) ____________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $3\,x_1+6\,x_2+2\,x_3+2\,x_4=-4$

$\quad x_1+2\,x_2+x_3+x_4=-1$

$\quad 6\,x_1+12\,x_2+5\,x_3+6\,x_4=-5$

a) Write down the coefficient matrix **A**, the RHS matrix $\vec{\mathbf{b}}$ and the augmented matrix $\mathbf{C} = \langle \mathbf{A} \mid \vec{\mathbf{b}} \rangle$ for this linear system of equations.

b) With technology (identify your choice!), reduce this matrix **C** step by step to its ReducedRowEchelonForm avoiding fractions (7 steps!), recording the intermediate matrices and row operations for each step (as in $R_1 \leftrightarrow R_2$, $R_3 \rightarrow R_3+2\,R_1$, $R_1 \rightarrow \frac{1}{2}\,R_1$). You may combine the AddRow operations with a single pivot, reporting only the final matrix.

c) Write out the equations that correspond to the reduced matrix. Identify the leading variables and the free variables and solve. State your solution in the scalar form: $x_1 = \ldots,\ x_2 = \ldots$, etc.

d) Does it agree with the technology solution of the original system? If not, there must be a mistake somewhere.

2. Use technology to solve the system (recall the instruction to identify your choice of technology, and perhaps you should rewrite these equations first lining up the variables):

$x_1-x_3=0$

$2\,x_1-x_4=0$

$5\,x_1-x_3=8$

$2\,x_2-x_3-2\,x_4=4$

This has an integer solution. Check that it is correct (by backsubstitution).

▶ solution

a) $A=\begin{bmatrix}3&6&2&2\\1&2&1&1\\6&12&5&6\end{bmatrix}$, $\vec{b}=\begin{bmatrix}-4\\-1\\-5\end{bmatrix}$, $C=\begin{bmatrix}3&6&2&2&-4\\1&2&1&1&-1\\6&12&5&6&-5\end{bmatrix}$

b) $\xrightarrow{R_1\leftrightarrow R_2}\begin{bmatrix}1&2&1&1&-1\\3&6&2&2&-4\\6&12&5&6&-5\end{bmatrix}\xrightarrow[R_3\to R_3-6R_1]{R_2\to R_2-3R_1}\begin{bmatrix}1&2&1&1&-1\\0&0&-1&-1&-1\\0&0&-1&0&1\end{bmatrix}\xrightarrow{R_2\to -R_2}\begin{bmatrix}1&2&1&1&-1\\0&0&1&1&1\\0&0&-1&0&+1\end{bmatrix}$

$\xrightarrow[R_3\to R_3+R_2]{R_1\to R_1-R_2}\begin{bmatrix}1&2&0&0&-2\\0&0&1&1&1\\0&0&0&1&2\end{bmatrix}\xrightarrow{R_2\to R_2-R_3}\begin{array}{c}\begin{bmatrix}1&2&0&0&-2\\0&0&1&0&-1\\0&0&0&1&2\end{bmatrix}\\ \begin{matrix}x_1&x_2&x_3&x_4\\L&F&L&L\end{matrix}\end{array}$

c) $x_1+2x_2=-2$, $x_3=-1$, $x_4=2$ $\xrightarrow{x_2=t}$ $x_1=-2-2t$, $x_3=-1$, $x_4=2$

$\boxed{\langle x_1,x_2,x_3,x_4\rangle=\langle -2-2t,t,-1,2\rangle}$ or $\boxed{x_1=-2-2t,\ x_2=t,\ x_3=-1,\ x_4=2}$

d) yes, Maple gives exactly this solution.

(2) $x_1+0x_2-x_3+0x_4=0$

$2x_1+0x_2+0x_3-x_4=0$

$5x_1+0x_2-x_3=8$

$0x_1+2x_2-x_3-2x_4=4$

Augmented Matrix: $\begin{bmatrix}1&0&-1&0&0\\2&0&0&-1&0\\5&0&-1&0&8\\0&2&-1&-2&4\end{bmatrix}\xrightarrow[\text{maple}]{\text{rref}}\begin{array}{c}\begin{matrix}L&L&L&L\\x_1&x_2&x_3&x_4\end{matrix}\\ \begin{bmatrix}1&0&0&0&2\\0&1&0&0&7\\0&0&1&0&2\\0&0&0&1&4\end{bmatrix}\end{array}$

solution: $\boxed{x_1=2,\ x_2=7,\ x_3=2,\ x_4=4.}$

check: $(2)-(2)=0 \rightarrow 0=0$ ✓

$2(2)-(4)=0 \rightarrow 4-4=0 \rightarrow 0=0$ ✓

$5(2)-(2)=8 \rightarrow 10-2=8 \rightarrow 8=8$ ✓

$2(7)-(2)-2(4)=4 \rightarrow 14-2-8=4 \quad 4=4$ ✓