MAT2705-01/02 14F Test 1 Print Name (Last, First) _____|__

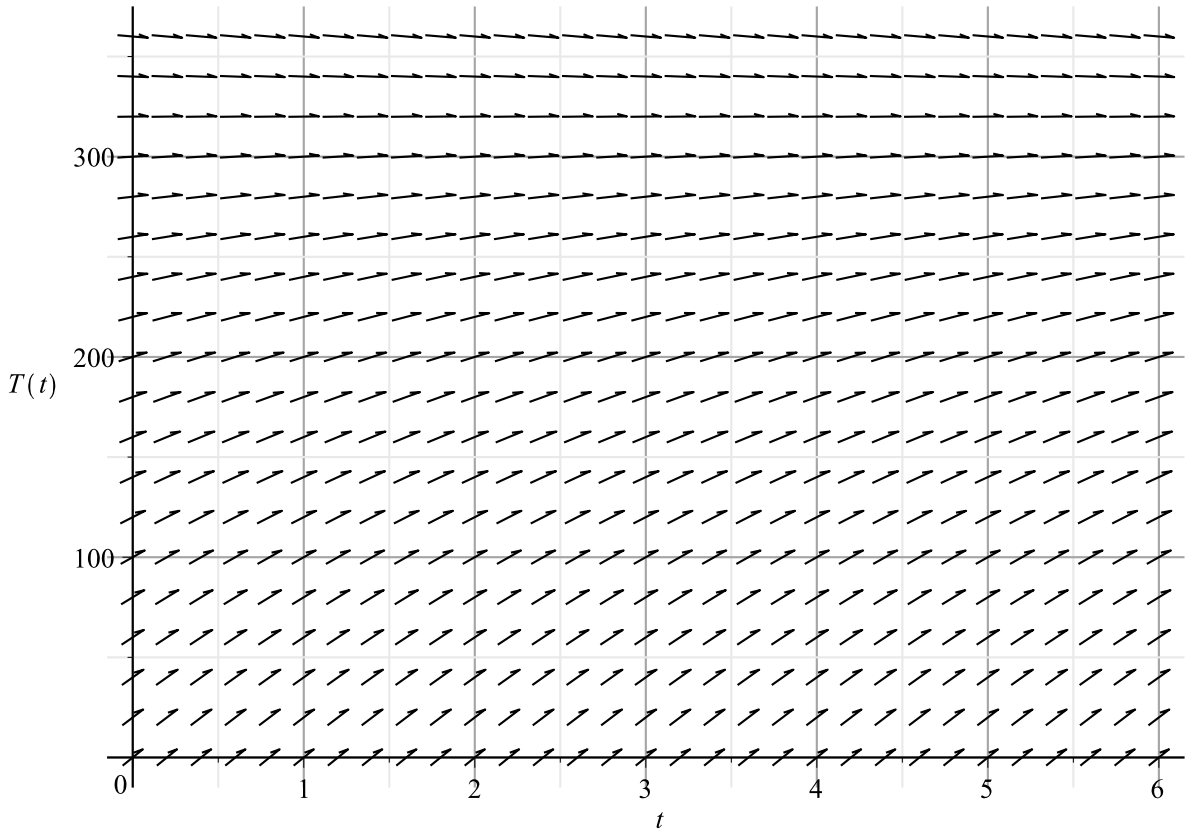
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). [Recall you need $y'(t), y(t)$ instead of y', y in your differential equation for an unknown variable y for Maple to interpret the prime as a t derivative.]

1. $\frac{dT}{dt} = -k\,(T - 325)$

a) Bob pulls a turkey from the refrigerator at 50 degrees and jams it into a 325 degree oven. After 1.5 hours the instant read thermometer jabbed into the turkey thigh reaches 125 degrees. If this turkey thigh obeys Newton's law of cooling/heating, how long does bob have to wait till the minimum recommended temperature of 165 degrees is reached? [Use the separable solution technique to solve this DE. Show every step of the process clearly. Answer this word problem with a complete English sentence which can be directly compared to a clock. Boxit. During the process keep things exact using 3/2 for 1.5 and don't introduce any decimal points long enough for you reach exact values for k and τ requested below.]

b) Using the slope field below, locate the initial data point and the secondary data point by circled dots. Then make a rough hand sketch of your solution, labeling on your sketch the initial and secondary data points given above, and including the horizontal lines corresponding to 325, 165 and 125 degrees (use a piece of paper as a straight edge?). Is your hand drawn curve consistent with your answer to the question posed in part a)? Explain.

c) What is the exact value of k and its numerical value to 6 decimal places? What is the exact value of the corresponding characteristic time τ (with units) and its numerical value to 3 significant digits? Mark this on the time axis and read off the temperature from your hand plot and compare with your predicted value at $t = \tau$. Comment.



(turn the page)

2. $2\,\frac{dx}{dt} = 10 - \frac{4}{9+4\,t}\,x, \quad x(0) = 10$

a) Solve this initial value problem with Maple and write down the solution.
[Use function notation to input the DE: $2\ x'(t) = ...\ x(t)$].
b) Put this differential equation into the standard form for a first order linear differential equation.
c) Evaluate the integrating factor.
d) Proceed to solve the differential equation to state the general solution for $x(t)$.
e) Impose the initial condition on this solution.
f) What is the exact and approximate value of $x(4)$?
g) Check that your IVP solution of the DE is correct by substitution and simplification.
[You can avoid some algebra by not multiplying out the factor $9 + 4\,t$ which appears in your solution before checking.
]

optional (ignore this unless you finish early and are confident everything you have done is correct):

h) If this is a mixing problem, the concentration of solute leaving the tank is $c(t) = \frac{x(t)}{9+4\,t}$. Evaluate $c(0)$ and $c(4)$.

When does this concentration equal 1?

Be sure to sign and date the pledge before handing in this test.

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants.
"

Signature: Date: