MAT2500-01 13S Quiz 8   Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. Consider the integral $\int_0^2 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} (x^2+y^2)\, y^2 \, dy \, dx$.

1 a) Evaluate this iterated integral exactly (no decimals) with technology.

3 b) Deconstruct this integral, identifying its bounding curves, and make a diagram "shading in" the region of integration, and showing a typical cross-section with its directional arrow indicating the inner integration, labeling its endpoints properly. [Label axes, tickmarks, intercepts, etc.].

3 c) Now re-express the bounding curves as a single quadratic condition and covert it to polar coordinates, solving for the radial variable, and then make a similar new diagram indicating the corresponding situation for polar coordinate integration, showing the typical radial cross-section with properly labeled endpoints (arrow in direction of increasing values of the radial coordinate). State the range of *r* values in terms of θ.

2 d) What is the angular range for θ? Explain.

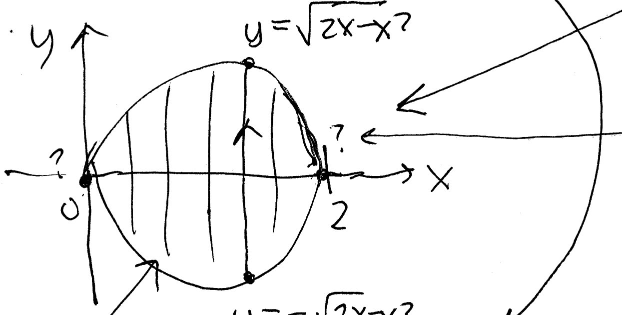
1 e) Write down the new iterated double integral and evaluate it using technology. Do you get the correct result? If not, can you find your error in setup?

▶ solution

b) $\int_{x=0}^{x=2} \int_{y=-\sqrt{2x-x^2}}^{y=\sqrt{2x-x^2}} (x^2+y^2)\, y^2 \, dy \, dx$ (a) $\underset{\text{Maple}}{=} \frac{5\pi}{12}$

c) $y^2 = 2x - x^2 = x(2-x) = 0$ at $x=0,2$

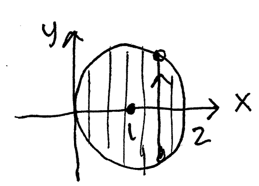
$x^2+y^2=2x$



? means with knowing it is a circle we dont know how top & bot graphs meet.

aha, is a circle: must have center on x axis at x=1 dont even have to complete square to see this because symmetric across x-axis

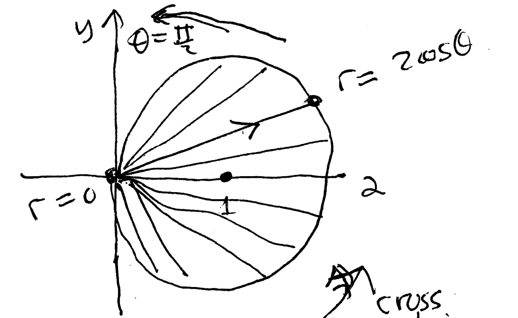
better plot:



c) $x^2+y^2=2x$

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$, $r=0$



d) $\theta = -\frac{\pi}{2}$

cross sections sweep out this interval

$\boxed{\begin{array}{l} \text{c) } r = 0..2\cos\theta \\ \text{d) while } \theta = -\frac{\pi}{2}..\frac{\pi}{2} \end{array}}$

e) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} \underbrace{(r^2)}_{x^2+y^2} \underbrace{(r\sin\theta)^2}_{y^2} \underbrace{r\,dr\,d\theta}_{dA}$

$= \boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} r^5 \sin^2\theta \, dr\, d\theta}$

$= \boxed{\frac{5\pi}{12}}$ ✓ note: $\left.\frac{r^6}{6}\right|_0^{2\cos\theta} = \frac{32}{3}\cos^6\theta$

but $\int \cos^6\theta \sin^2\theta \, d\theta$ needs technology