MAT2500-01/04 TEST 2 Answers (1)

① $f(x,y) = \frac{5x}{x^2+y^2} = 5x(x^2+y^2)^{-1}$, $f(1,2) = \frac{5}{1+4} = 1$

a) $f_x = \frac{5[\cancel{(x^2+y^2)(1) - x(2x)}]}{(x^2+y^2)^2} = \frac{5(y^2-x^2)}{(x^2+y^2)^2}$

$f_y = 5x(-1)(x^2+y^2)^{-2}(2y = \frac{-10xy}{(x^2+y^2)^2}$

$\vec{\nabla} f(x,y) = \frac{5\langle y^2-x^2, -2xy\rangle}{(x^2+y^2)^2}$

$\vec{\nabla} f(1,2) = \frac{5\langle 4-1, -4\rangle}{5^2} = \frac{\langle 3,-4\rangle}{5}$

$|\vec{\nabla} f(1,2)| = \frac{1}{5}\sqrt{3^2+4^2} = \frac{5}{5} = 1$ unit vector.

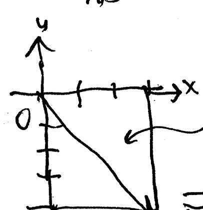
$\boxed{\hat{u} = \langle \frac{3}{5}, \frac{-4}{5}\rangle = \frac{\langle 3,-4\rangle}{5}}$ gives direction of most rapid increase.

b) $\vec{v} = \langle 2,-1\rangle$, $\hat{v} = \frac{\langle 2,-1\rangle}{\sqrt{5}}$

$D_{\hat{v}} f(1,2) = \hat{v}\cdot\vec{\nabla} f(1,2)$

$= \frac{\langle 2,-1\rangle}{\sqrt{5}} \cdot \frac{\langle 3,-4\rangle}{5} = \frac{10}{5\sqrt{5}} = \boxed{\frac{2}{\sqrt{5}}}$

c)



$m = \boxed{-\frac{4}{3} = \text{slope}}$

normal line is along gradient.

$f(1,2) = \frac{5(1)}{1+4} = 1$

$\boxed{\frac{5x}{x^2+y^2} = 1 \quad \text{or} \quad x^2+y^2 = 5x}$

circle with center on x-axis.

d) $\boxed{L(x,y)} = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$

$\boxed{= 1 + \frac{3}{5}(x-1) - \frac{4}{5}(y-2)}$

e) $f(.98, 2.01) \approx L(.98, 2.01) = 1 + \frac{3}{5}(.98-1) - \frac{4}{5}(2.01-2)$

$= 1 + \frac{3}{5}(-.02) - \frac{4}{5}(.01) = 1 \; \frac{-.04 - .06}{5} =$

$= 1 - .02 = \boxed{0.98}$

② $f(x,y) = x^3 - 3x - y^3 + 12y$

a) $f_x = 3x^2 - 3 = 3(x^2-1) = 0 \to x^2 = \pm 1$

$f_y = -3y^2 + 12 = 3(-y^2 + \;) = 0 \to y^2 = \pm 2$

$f_{xx} = 6x$

$f_{yy} = -6y$

$f_{xy} = 0$

$f_x(1,2) = 3(1-1) = 0$ ✓ $f_x(-1,2) = 3(1-1) = 0$ ✓

$f_y(1,2) = 3(4-4) = 0$ ✓ $f_y(-1,2) = 3(4-4) = 0$ ✓

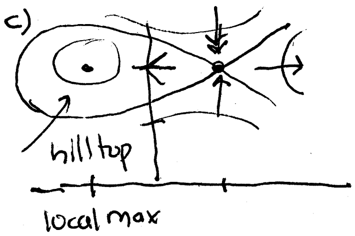
(1,2), (−1,2) are critical pts.

b)

	(1,2)		(−1,2)	
f_{xx}	6	∪	−6	∩
f_{yy}	−12	∩	−12	∩
f_{xy}	0	saddle	0	
$f_{xx}f_{yy} - f_{xy}^2$	$-72 < 0$ saddle		$72 > 0$	

} local max? ↑ confirms local max.

c)



increases in arrow directions so saddle.

③ $\frac{\partial}{\partial z}(yz + x\ln y = z^2 + 1)$ $y = y(x,z)$ dep var.

$\frac{\partial y}{\partial z} z + y(1) + x\frac{1}{y}\frac{\partial y}{\partial z} = 2z$, $\frac{\partial y}{\partial z}(\frac{x}{y} + z) = 2z - y$

$\frac{\partial y}{\partial z} = \frac{2z-y}{\frac{x}{y}+z} = \frac{y(2z-y)}{x+yz}$ $\left.\frac{\partial y}{\partial z}\right|_{(1,e,e)} = \frac{e(2e-e)}{1+e^2}$

$= \boxed{\frac{e^2}{1+e^2}}$

④ $F(x,y,z) = \frac{x^2}{4} + y^2 + \frac{z^2}{3} = 5$

a) $\vec{\nabla} F = \langle \frac{x}{2}, 2y, \frac{2}{3}z\rangle$, $\vec{\nabla} F(2,1,-3) = \boxed{\langle -1, 2, -2\rangle = \vec{n}}$

$0 = \vec{n}\cdot(\vec{r} - \vec{r}_0) = \langle -1,2,-2\rangle \cdot \langle x+2, y-1, z+3\rangle$

$= -(x+2) + 2(y-1) - 2(z+3)$

$= -x - 2 + 2y - 2 - 2z - 6$

$= \boxed{-x + 2y - 2z - 10 = 0}$

$\boxed{\text{or} \quad x - 2y + 2z = -10}$

b) $\vec{r} = \vec{r}_0 + t\vec{n}$

$\langle x,y,z\rangle = \langle -2,1,-3\rangle + t\langle -1,2,-2\rangle$

$= \langle -2-t, 1+2t, -3-2t\rangle$ normal line

OPTIONAL:

④ c): $\frac{(-2-t)^2}{4} + (1+2t)^2 + \frac{(-3-2t)^2}{3} = 5$

↳ maple: $t = 0, \boxed{-\frac{108}{67}}$ ↳ subs in $\vec{r}(t)$: $\vec{r}(-\frac{108}{67}) = \frac{1}{67}\langle -26, -149, 15\rangle$ ✓

MAT2500-01 13S Quiz 8 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. Consider the integral $\int_0^2 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} (x^2+y^2)\, y^2 \, dy\, dx$.

1 a) Evaluate this iterated integral exactly (no decimals) with technology.

3 b) Deconstruct this integral, identifying its bounding curves, and make a diagram "shading in" the region of integration, and showing a typical cross-section with its directional arrow indicating the inner integration, labeling its endpoints properly. [Label axes, tickmarks, intercepts, etc.].

3 c) Now re-express the bounding curves as a single quadratic condition and covert it to polar coordinates, solving for the radial variable, and then make a similar new diagram indicating the corresponding situation for polar coordinate integration, showing the typical radial cross-section with properly labeled endpoints (arrow in direction of increasing values of the radial coordinate). State the range of *r* values in terms of θ.

2 d) What is the angular range for θ? Explain.

1 e) Write down the new iterated double integral and evaluate it using technology. Do you get the correct result? If not, can you find your error in setup?

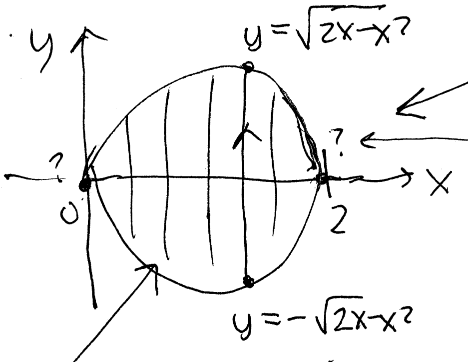
▶ solution

b) $\int_{x=0}^{x=2} \int_{y=-\sqrt{2x-x^2}}^{y=\sqrt{2x-x^2}} (x^2+y^2)\, y^2 \, dy\, dx$

(a) $\underset{\text{Maple}}{=} \frac{5\pi}{12}$

c) $y^2 = 2x - x^2 = x(2-x) = 0$ at $x=0,2$

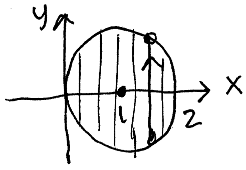
$x^2+y^2=2x$



? means with knowing it is a circle we dont know how top & bot graphs meet.

aha, is a circle: must have center on X axis at X=1 dont even have to complete square to see this because symmetric across X-axis

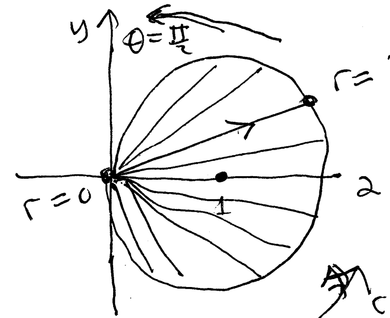
better plot:



c) $x^2+y^2 = 2x$

$r^2 = 2r\cos\theta$

$r = 2\cos\theta\ ,\ r=0$



d) $\theta = -\frac{\pi}{2}$

cross sections sweep out this interval

$\boxed{\begin{array}{l} \text{c) } r = 0..2\cos\theta \\ \text{d) while } \theta = -\frac{\pi}{2}..\frac{\pi}{2} \end{array}}$

e) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} \underbrace{(r^2)}_{x^2+y^2} \underbrace{(r\sin\theta)^2}_{y^2} \underbrace{r\,dr\,d\theta}_{dA}$

$= \boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} r^5 \sin^2\theta \, dr\, d\theta}$

$= \boxed{\frac{5\pi}{12}}$ ✓ note: $\left.\frac{r^6}{6}\right|_0^{2\cos\theta} = \frac{32}{3}\cos^6\theta$

but $\int \cos^6\theta \sin^2\theta \, d\theta$ needs technology

MAT2500-01 13S Quiz 7 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of each problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. $V=\iiint_R 1\ dV$; R is the region of the first octant bounded by the coordinate planes $x=0$, $y=0$, $z=0$, the cylinder $x=4-y^2$ and the plane $y+z=2$ as shown in the figure below.

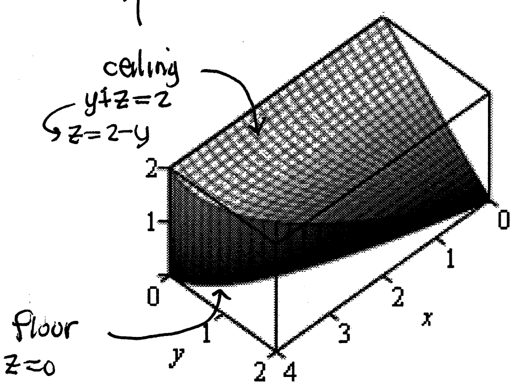
a) Iterate this triple integral in Cartesian coordinates in the order $\iiint \ldots\, dz\, dx\, dy$, then use Maple to evaluate the result exactly (don't use decimal numbers anywhere, although your result to one decimal place should be 6.7 as a check on your iteration of the integral). Support your result with a diagram of the outer double integral region of integration in the xy-plane showing one typical line segment cross-section with its endpoints labeled, and "shade in" the region with equally spaced such cross-section line segments, while stating the floor and ceiling function graphs which bound the region in the z direction.

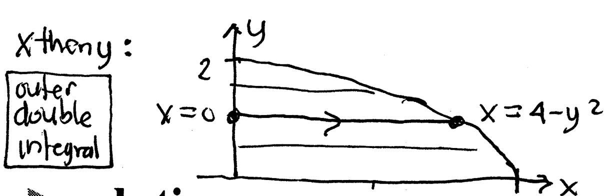
b) Re-express the integral in the order $\iiint 1\, dx\, dy\, dz$ and evaluate by hand this triple integral. Support your result with a diagram of the region of integration in the yz-plane showing one typical cross-section with its endpoints labeled, and "shade in" the region with equally spaced cross-section line segments. Does your result agree with part a)?

c) Now use Maple to evaluate the z-component of the centroid of the solid: $Z=\dfrac{\left(\iiint_R z\,dV\right)}{\iiint_R 1\ dV}$. What is its numerical approximation? Should the geometric center (centroid) be above or below the plane $z=1$ for this solid, given where most of its volume lies? Does your result agree with your reasoning?

a) $\boxed{\text{inner integral}}$

z first: $z = 0 .. y-2$



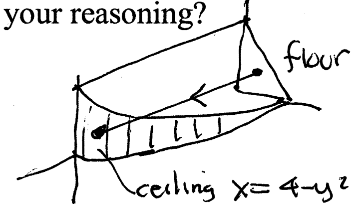


▶ **solution**

$x=0 .. 4-y^2$ while $y=0..2$

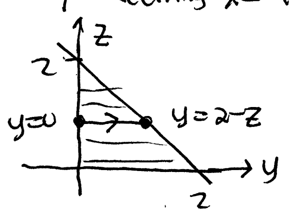
$$\boxed{\int_0^2\int_0^{4-y^2}\int_0^{y-2} 1\ dz\,dx\,dy} \overset{\text{Maple}}{=} \frac{20}{3} \approx 6.6667 \checkmark$$

b)



$\boxed{\text{innermost integral}}$

x first: $x=0..4-y^2$



$\boxed{\text{outer double integral}}$

$y=0..2-z$ while $z=0..2$

$$\boxed{\int_0^2\int_0^{2-z}\int_0^{4-y^2} 1\ dx\,dy\,dz}$$

$$= \int_0^2\int_0^{2-z} \underbrace{x\Big|_{x=0}^{x=4-y^2}}_{4-y^2-0} dy\,dz = \int_0^2 4y-\frac{y^3}{3}\Big|_{y=0}^{y=2-z} dz$$

$$= \int_0^2 4(\underbrace{2-z}_{u})-\frac{(2-z)^3}{3}\,dz = \int\left(4u-\frac{u^3}{3}\right)(-du)$$

$du=-dz$

$$= -\frac{4u^2}{2}+\frac{u^4}{12} = -2(2-z)^2+\frac{(2-z)^4}{12}\Big|_0^2$$

$$= 0 + 2(2^2) - \frac{2^4}{12} = 8\cancel{\left(1-\tfrac{1}{6}\right)=\tfrac{64}{9}}$$

$$= 8 - \frac{4}{3} = \boxed{\frac{20}{3}} \checkmark$$

c) $\dfrac{\iiint z\,dV = \frac{24}{5}}{\iiint 1\,dV = \frac{20}{3}}$ $\quad Z = \dfrac{24}{5}\cdot\dfrac{3}{20} = \boxed{\dfrac{18}{25}}$

$\boxed{\approx 0.72}$

should be below $z=1$ & is !!