MAT2500-01 13S Quiz 7 Print Name (Last, First) ____________________________________|__

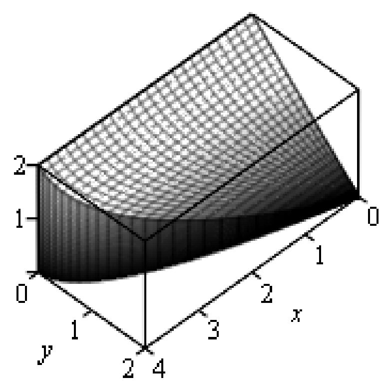
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of each problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. $V=\iiint_R 1\ \mathrm{d}V$; R is the region of the first octant bounded by the coordinate planes $x=0, y=0, z=0$, the cylinder $x=4-y^2$ and the plane $y+z=2$ as shown in the figure below.

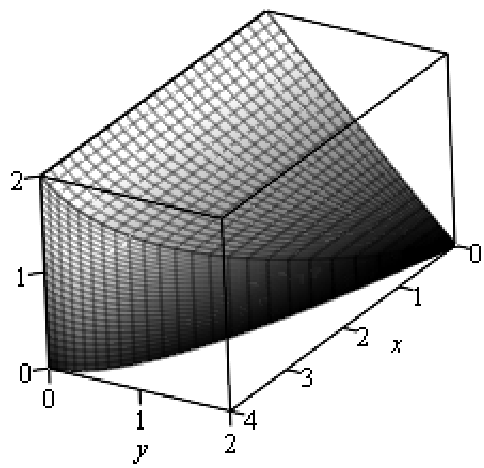
a) Iterate this triple integral in Cartesian coordinates in the order $\iiint \ldots\, dz\, dx\, dy$, then use Maple to evaluate the result exactly (don't use decimal numbers anywhere, although your result to one decimal place should be 6.7 as a check on your iteration of the integral). Support your result with a diagram of the outer double integral region of integration in the xy-plane showing one typical line segment cross-section with its endpoints labeled, and "shade in" the region with equally spaced such cross-section line segments, while stating the floor and ceiling function graphs which bound the region in the z direction.

b) Re-express the integral in the order $\iiint 1\ dx\, dy\, dz$ and evaluate by hand this triple integral. Support your result with a diagram of the region of integration in the yz-plane showing one typical cross-section with its endpoints labeled, and "shade in" the region with equally spaced cross-section line segments. Does your result agree with part a)?

c) Now use Maple to evaluate the z-component of the centroid of the solid: $Z=\dfrac{\left(\iiint_R z\ \mathrm{d}V\right)}{\iiint_R 1\ \mathrm{d}V}$. What is its numerical approximation? Should the geometric center (centroid) be above or below the plane $z=1$ for this solid, given where most of its volume lies? Does your result agree with your reasoning?



▶ solution



$x = 4 - y^2$ and the plane $y + z = 2$