MAT2500-01 13s Quiz 4 Take Home Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. $f(x,y) = \dfrac{xy}{x-y}$

a) Evaluate f and the first partial derivatives of f at $(x,y) = (2,1)$ using proper notation for all derivatives evaluated in the process.

b) Evaluate $f_{xy}(x,y)$ and its value at $(2,1)$.

2. If $P(L,K) = 3\,L^{\frac{2}{3}}\,K^{\frac{1}{3}}$, evaluate all the second derivatives, and then simplify the expression $\dfrac{\partial^2 P}{\partial L^2}\,\dfrac{\partial^2 P}{\partial K^2} - \left(\dfrac{\partial^2 P}{\partial L\,\partial K}\right)^2$.

3. The wave heights $h = h(v,t)$ (feet) in the open sea depend on the speed v (knots) of the wind and the length of time t (hours) that the wind has been blowing at that speed. Translate the mathematical condition $9 = h(30,5)$ into a complete English sentence.

▶ solution

1 a) $f = \dfrac{xy}{x-y}$

$$f_x = \frac{(x-y)\frac{\partial}{\partial x}(xy) - xy\frac{\partial}{\partial x}(x-y)}{(x-y)^2}$$

$$= \frac{(x-y)\,y - xy\,(1)}{(x-y)^2} = \frac{xy - y^2 - xy}{(x-y)^2}$$

$$= \boxed{\frac{-y^2}{(x-y)^2}}$$

$$f_y = \frac{(x-y)\frac{\partial}{\partial y}(xy) - xy\frac{\partial}{\partial y}(x-y)}{(x-y)^2}$$

$$= \frac{(x-y)(x) - xy(-1)}{(x-y)^2} = \frac{x^2 - xy + xy}{(x-y)^2}$$

$$= \boxed{\frac{x^2}{(x-y)^2}}$$

$$f(2,1) = \frac{2(1)}{2-1} = \frac{2}{1} = \boxed{2}$$

$$f_x(2,1) = \frac{-1^2}{(2-1)^2} = \boxed{-1}$$

$$f_y(2,1) = \frac{2^2}{(2-1)^2} = \boxed{4}$$

$$f_{xy} = (f_x)_y = \frac{\partial}{\partial y}\left(\frac{-y^2}{(x-y)^2}\right) = -\left(\frac{(x-y)^2\overbrace{\tfrac{\partial}{\partial y}(y^2)}^{2y} - y^2\overbrace{\tfrac{\partial}{\partial y}(x-y)^2}^{2(x-y)(-1)}}{(x-y)^4}\right)$$

$$= -\frac{[x-y][(x-y)(2y) + 2(y^2)]}{(x-y)^4} = -\frac{2xy - \cancel{2y^2} + \cancel{2y^2}}{(x-y)^3} = \boxed{\frac{-2xy}{(x-y)^3}}$$

$$f_{xy}(2,1) = \frac{-2(2)(1)}{(2-1)^3} = \boxed{-4}$$

2. $P = 3L^{2/3}K^{1/3}$

$$P_L = 3\left(\tfrac{2}{3}L^{-1/3}\right)K^{1/3} = 2K^{1/3}/L^{1/3} = 2L^{-1/3}K^{1/3}$$

$$P_K = 3L^{2/3}\left(\tfrac{1}{3}K^{-2/3}\right) = L^{2/3}K^{-2/3}$$

$$P_{LL} = 2\left(-\tfrac{1}{3}L^{-4/3}\right)K^{1/3} = \boxed{-\tfrac{2}{3}L^{-4/3}K^{1/3}}$$

$$P_{KK} = L^{2/3}\left(-\tfrac{2}{3}K^{-5/3}\right) = \boxed{-\tfrac{2}{3}L^{2/3}K^{-5/3}}$$

$$P_{LK} = 2L^{-1/3}\left(\tfrac{1}{3}K^{-2/3}\right) = \boxed{\tfrac{2}{3}L^{-1/3}K^{-2/3}}$$

$$P_{LL}P_{KK} - P_{LK}^2 = \left(-\tfrac{2}{3}L^{-4/3}K^{1/3}\right)\left(-\tfrac{2}{3}L^{2/3}K^{-5/3}\right) - \tfrac{4}{9}L^{-2/3}K^{-4/3}$$

$$= \tfrac{4}{9}L^{-2/3}K^{-4/3} - \tfrac{4}{9}L^{-2/3}K^{-4/3} = \boxed{0}!$$

3. If the wind has been blowing at 30 knots for 5 hours, the waves should be 9 feet high.