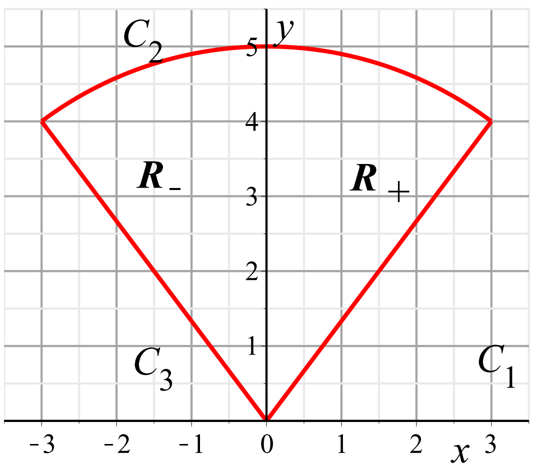
MAT2500-01 13S Final Exam   Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Use technology to check any integrals you set up.

1. Given the point $(x, y, z) = (-6, -8, -10)$, find the new coordinates, in each case stating the angles both in radians (exactly, using inverse trig functions) and in degrees (1 decimal place accuracy) and use proper identifying symbols for all coordinates: a) cylindrical coordinates. b) spherical coordinates. Support your work with two diagrams, one of the *x-y* plane and one of the *r-z* half plane, each including a reference triangle locating the point with respect to the axes with all three sides labeled by their lengths and both axes labeled by their coordinate labels and showing the relevant angles. Show clearly how you obtain values of your coordinates from these diagrams. Do the angles look right in these diagrams?



2.a) Describe the 3 curve segments comprising the counterclockwise directed curve C, first in Cartesian coordinates as function graphs (y as a function of x), the middle segment being an arc of a circle centered at the origin. Then describe them in terms of conditions expressed in polar coordinates. [Remember, you can use inverse trig functions to express angles exactly.] In each case state the range of the independent variable along each curve segment using inequalities.

b) What is the exact opening angle $\Delta\theta$ of this sector of the circle enclosed by the curve C and what is its approximate equivalent in degrees? Evaluate the area of the sector $A = \frac{1}{2}\, a^2 \Delta\theta$, where a is the radius of the circle.
Approximate this to 4 decimal places.

c) Let R_+ be the right half of this region R enclosed by C. Set up a Cartesian coordinate iteration of the integral $A = 2\iint_{R_+} 1 \, dA$ giving the area of the full region R as twice the area of its right half, and evaluate it exactly using technology, then approximately to 4 decimal places. Support your iteration with a diagram showing a typical bullet-endpoint line segment cross-section representing the inner integral with properly labeled endpoints and shading the region with equally spaced cross-sections.

d) Set up a polar coordinate iteration of the integral $A = \iint_R 1 \, dA$ over the full region and evaluate it exactly by hand, then approximately to 4 decimal places. Support your iteration with a similar appropriate diagram for these coordinates, as described above for the previous case.

e) Evaluate an iterated integral for $A_y = \iint y \, dA$ by hand using polar coordinates and evaluate exactly and numerically the ratio $\overline{y} = A_y / A$ giving the location of the centroid of the region on the vertical axis. Clearly mark it on the test sheet diagram.

f) Evaluate the line integral of the vector field $\vec{F} = \langle -y, x\rangle /2$ around C.

g) Green's theorem states that $\int_C \vec{F} \cdot d\vec{r} = \iint_R \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dA$. Evaluate the right hand side for this vector field. Compare with previous results.

3. Consider the sphere $x^2 + y^2 + (z-1)^2 = 1$ passing through the origin but not centered at the origin.
a) First express this condition in cylindrical coordinates. Make a labeled diagram of the *r-z* half plane showing the circular cross-section of this sphere. Where is the center of the sphere in this diagram?

b) Express the equation for this sphere in spherical coordinates and indicate a typical labeled cross-section in the *r-z* half plane showing the radial integration along ρ, and indicate the range of values of the polar angle ϕ.

c) Set up the integral $V = \iiint 1\, dV$ in the usual spherical coordinates (ρ, ϕ, θ) over this sphere to evaluate its volume. Then repeat to find the z-coordinate of its obvious center $\bar{z} = \frac{1}{V}\iiint z\, dV$ on the z-axis.

4. $\vec{F} = \langle x, -y, z\rangle$
a) Show that $\vec{F}$ satisfies the curlfree condition that it admit a potential function, i.e., is a conservative vector field.

b) Find a potential function f for it.

c) Use the potential to evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ over any curve from $P(1, 2, 3)$ to $Q(3, 1, 2)$. [If you want a check on this result, you can always do the line integral directly on a straight line segment.]

▶ solution (on-line)

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in with your answer sheets as a cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants. "

Signature: Date: