

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Explain in as many words as possible everything you are doing! For each hand integration step,

state the antiderivative formula used before substituting limits into it: $\int_a^b f(x) dx = F(x)|_{x=a}^{x=b} = F(b) - F(a)$.

1. $\int_0^{\frac{3}{2}} \int_{\sqrt{3}y}^{\sqrt{9-y^2}} xy dx dy$.

- Do this integral by hand.
- Make a completely labeled (shaded) diagram of the region of integration for this integral, with a typical labeled cross-section line segment (bullet endpoints) representing the current iteration of the integral.
- Redo this diagram appropriate for a polar coordinate iteration of the integral, indicating the appropriate range for the polar angle on the diagram.
- Evaluate the new integral by hand.

2. A rad integral $\int_0^1 \int_{2-2\sqrt{1-x}}^{2\sqrt{x}} y dy dx$ is an iterated integral over a region between two curves in the first quadrant.

- Evaluate this exactly using technology and then numerically evaluate the result.
- Make a completely labeled (shaded) diagram of the region of integration for this integral, with a typical labeled cross-section line segment (bullet endpoints) representing the current iteration of the integral. You may use technology to find the intersection points between the two boundary curves, if necessary.
- Make a new completely labeled diagram corresponding to the reversed order of integration.
- State the new integral with the order of integration reversed.
- Evaluate the new integral step by step exactly by hand and using technology.
- Do you get the same result as in part a)? If not find your error.

3. Consider the solid region R in the first octant corresponding to the region of integration in the triple integral

$\int_0^4 \int_{\sqrt{x}}^2 \int_0^{1-\frac{y}{2}} f(x, y, z) dz dy dx$. See the figure on page 2 (left).

- Make labeled plane diagrams of the projection of R onto the x - y plane (with a labeled cross-section for the inner integration of the outer double integral) and the x - z plane (with a labeled cross-section for the innermost integral), in each case labeling the cross-section by the starting and stopping value equations for the variable of integration.
- Rewrite the integral in the order $\iiint \dots dx dz dy$, supporting your outer limits of integration with a new labeled diagram as in part a).
- Rewrite the integral in the order $\iiint \dots dy dz dx$, supporting your limits of integration with 2 labeled diagrams as in part a).
- Evaluate all 3 integrals exactly by hand step by step for $f(x, y, z) = 1$ to get the volume of this region. Your results should agree.

4. Birdbath with center spike. Consider the solid of revolution R inside the lower hemisphere $x^2 + y^2 + z^2 = 4$ but outside the curve obtained by revolving the semicircle $(r - 1)^2 + z^2 = 1$ around the vertical axis, a vertical cross-section of which is given in the figure (next page, but note $r \geq 0$ only, while the diagram shows this cross-section revolved around the vertical axis as well).

- Write down an iterated integral for the volume of this solid in cylindrical coordinates, supporting your work with a new r - z half-plane diagram illustrating a typical cylindrical coordinate vertical cross-section with a superimposed arrow for its direction, labeling its endpoints by the starting and stopping values of the vertical coordinate, and describe the range of values of the remaining coordinates over this region.

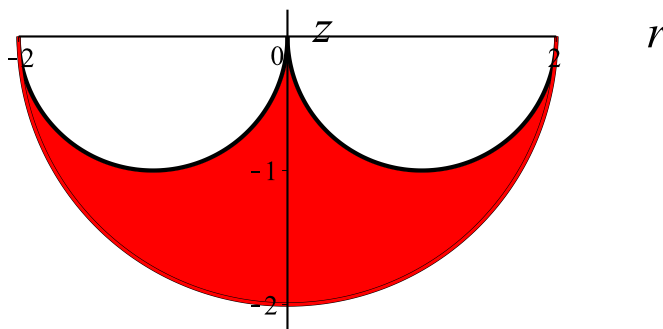
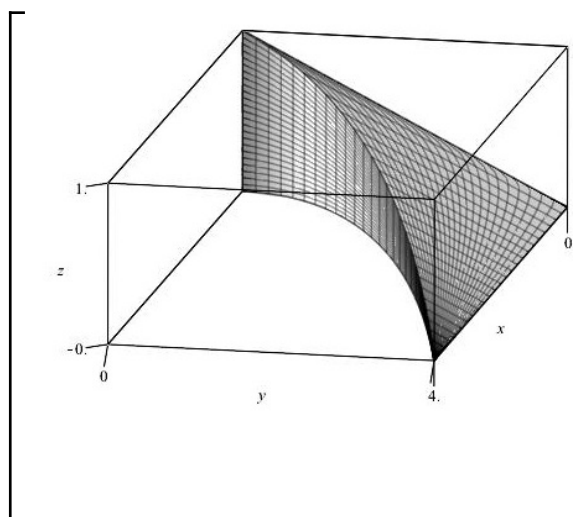
- b) Evaluate this integral exactly by hand step by step, using technology for the antiderivative at the second integration step.
- c) Repeat a new diagram for spherical coordinates, showing a typical radial cross-section with a superimposed arrow for its direction and labeling its endpoints by the starting and stopping values of the new radial coordinate and describe the range of values of the remaining coordinates over this region.
- e) From your new diagram write down an iterated triple integral in spherical coordinates representing the volume of this solid and evaluate it exactly by hand step by step.
- f) Do your two results agree with each other and the Maple evaluation of the triple integrals? If not find your error.
- g) The centroid (center of volume = center of mass for a homogeneous mass distribution with density function equal to 1) of this solid lies on the z axis because of the rotational symmetry about this axis. Choose one of these two coordinate systems to evaluate the z coordinate " \bar{z} " of the centroid exactly and numerically.
- h) Mark your centroid point on one of your diagrams, identifying it. Does this point seem reasonable to you? If it does not perhaps you made an error, so don't just automatically respond "yes."

5. a) Use spherical coordinates to represent $\int_0^5 \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \int_0^{\sqrt{25-x^2-y^2}} \frac{1}{1+x^2+y^2+z^2} dz dy dx$ as a triple integral,

supporting your limits of integration with labeled diagrams as above.

b) Evaluate this new integral exactly with technology and compare its value with the original triple integral in Cartesian coordinates evaluated with technology. Do they agree?

c) Evaluate the spherical coordinate integral exactly by hand step by step. Does it agree with your previous results?



No collaboration. You may only talk to bob. See test rules [on-line](#). Read short rules above. Print out and attach any Maple supporting work you do, hand annotating if necessary with problem number and part etc, whatever is necessary for clarification.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in stapled to your answer sheets as the cover page, with the Lastname, FirstName side face up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants.

"

Signature:

Date:

