MAT2500-01 13S TEST2 Answers

(1) a) $f(x,y) = x^2+x^2y+xy+xy^2$

$f_x = 2x+2xy+y+y^2$

$f_y = x^2+x+2xy$

$f_x(2,1) = 2(2)+2(2)(1)+1+1^2 = 10$

$f_y(2,1) = 2^2+2+2(2)(1) = 10$

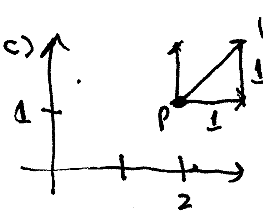
$\vec{\nabla} f(2,1) = \langle 10,10\rangle$

$\boxed{\hat{\nabla} f(2,1) = \frac{\langle 1,1\rangle}{\sqrt{2}}}$

b) $\vec{u} = \langle 1,-2\rangle$, $\hat{u} = \frac{\langle 1,-2\rangle}{\sqrt{5}}$

$D_{\hat{u}} f(2,1) = \hat{u}\cdot\vec{\nabla} f(2,1)$

$= \frac{\langle 1,-2\rangle}{\sqrt{5}}\cdot\langle 10,10\rangle = \boxed{-\frac{10}{\sqrt{5}}}$ $\boxed{<0 \text{ so decreasing}}$

c) 

$m = \frac{1}{1} = \boxed{1}$!

$f(2,1) = 2^2+2^2(1)+2(1)+2(1)^2$
$= 12$

contour thru P: $\boxed{x^2+x^2y+xy+xy^2=12}$

(2) a) $f(x,y) = -x^3+4xy-2y^2+1$

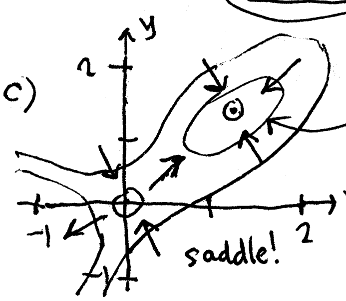
$f_x = -3x^2+4y = 0 \to -3x^2+4x = x(4-3x)=0$

$f_y = 4x-4y = 0 \to y=x$

$x=0$ $x=4/3$
$y=0$ $y=4/3$

$\boxed{\text{critical pts: } (0,0),\ (\tfrac{4}{3},\tfrac{4}{3})}$

	$(0,0)$	$(4/3, 4/3)$	
$f_{xx} = -6x$	0	$-6\cdot 4/3 = -8 < 0$	suggests local max
$f_{yy} = -4$	-4	$-4 < 0$	
$f_{xy} = 4$	4	4	
$f_{xx}f_{yy}-f_{xy}^2$	$-16<0$	$(-8)(-4)-4^2 = 16>0$	
	saddle	confirms local max!	

c) 

arrow indicates direction of increase

$\boxed{f_{xx}<0,\ D>0 \text{ reasoning is a lack of thinking — book memorized condition}}$

(3) $(x+\frac{4}{y^2})(y-1) = z$ $\frac{\partial y}{\partial z}, \frac{\partial y}{\partial x}$ ← dep ← ind

$\frac{\partial}{\partial z}\left[(x+4y^{-2})(y-1) = z\right]$

$(0+4(-2)y^{-3}\frac{\partial y}{\partial z})(y-1) + (x+4y^{-2})(\frac{\partial y}{\partial z}-0) = 1$

$-\frac{8(y-1)}{y^3}\frac{\partial y}{\partial z} + (x+4y^{-2})\frac{\partial y}{\partial z} = 1$

$\left[(x+4y^{-2}) - \frac{8(y-1)}{y^3}\right]\frac{\partial y}{\partial z} = 1$

$\boxed{\frac{\partial y}{\partial z} = \frac{1}{(x+4/y^2)-8(y-1)/y^3} = \frac{y^3}{xy^3-4y+8}}$ simplify!

$\left.\frac{\partial y}{\partial z}\right|_{(1,2,2)} = \frac{1}{(1+4/2^2)-8(2-1)/2^3} = \frac{1}{2-1} = \frac{1}{1} = \boxed{1}$

(4) a) $F(x,y,z) = 3x^{-1}+2y^{-1}+z^{-1}-xyz$ $\vec{r}_0 = \langle 1,1,-1\rangle$

$\vec{\nabla} F(x,y,z) = \langle -3x^{-2}-yz, -2y^{-2}-xz, -z^{-2}-xy\rangle$

$\vec{\nabla} F(1,1,-1) = \langle -3+1, -2+1, -1-1\rangle$
$= \langle -2,-1,-2\rangle = -\langle 2,1,2\rangle$ simplest

$\boxed{F \text{ increases along gradient!}}$

$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle 2,1,2\rangle\cdot\langle x-1,y-1,z+1\rangle$
$= 2(x-1)+(y-1)+2(z+1) = 2x+y+2z-2-1+2$

$\boxed{2x+y+2z=1}$

b) $\vec{r} = \vec{r}_0+t\vec{n} = \langle 1,1,-1\rangle + t\langle 2,1,2\rangle$

$\boxed{\langle x,y,z\rangle = \langle 1+2t, 1+t, -1+2t\rangle}$

$\boxed{\text{or } x=1+2t,\ y=1+t,\ z=-1+2t}$

Since we reversed the sign of the gradient as we increase t, we $\boxed{\text{decrease } F}$

c) $L(x,y,z) = F(1,1,-1)+F_x(1,1,-1)(x-1)$
$+F_y(1,1,-1)(y-1)+F_z(1,1,-1)(z+1)$

$\boxed{F(1,1,-1) = 3+2-1+1 = 5}$ $= 5-2(x-1)-(y-1)+2(z+1)$

$\boxed{L(x,y,z) = 5-2(x-1)-(y-1)-2(z+1)}$

d) $F(0.99, 1.02, -1.03)$
$\approx L(0.99, 1.02, -1.03)$
$= 5-2(0.99-1)-(1.02-1)-2(-1.03+1)$
$= 5+0.02-.02+.06 = \boxed{5.06}$