MAT2500-01 13S Test 2 Print Name (Last, First) _____|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type(Maple, GC). You are encouraged to use technology to check all of your hand results.

1. $f(x,y) = x^2 + x^2 y + x y + x y^2$; $P(2,1)$
a) Find a unit vector in the direction in which this function increases most rapidly at P.
b) Evaluate the directional derivative of this function at P in the direction parallel to the vector $\langle 1, -2 \rangle$. Is f increasing or decreasing in this direction?
c) What is the slope of the normal line to the contour passing through P ? What is the equation of this contour in simplest form?

2. $f(x,y) = -x^3 + 4 x y - 2 y^2 + 1$
a) Find the two critical points of f.
b) Use the second derivative test to classify both of these critical points as local minima, local maxima or saddle points. Use words to explain your reasoning.
c) Does a quick contourplot of this function confirm your conclusions? Explain. (Right click on function, plots, plot builder, use a small enough window or the option contours = 50).

3) If $\left(x + \frac{4}{y^2}\right)(y-1) = z$, find $\frac{\partial y}{\partial z}$ and evaluate it at the point $(1, 2, 2)$. [Use implicit differentiation! This is a special case of the van der Vaals generalization of the ideal gas equation, see Stewart Exercise 14.3.87.]

4. a) Find the equation of the tangent plane to the level surface of $F(x,y,z) = \frac{3}{x} + \frac{2}{y} + \frac{1}{z} - x y z$ at the point $P(1, 1, -1)$, and simplify it to the standard linear form. Identify a (simplest) normal vector $\vec{\boldsymbol{n}}$ to this plane.
b) What are the parametrized equations for the normal line to this surface at the same point using this normal? How does the function change as your parameter increases (increase/decrease? explain)?
c) Find the linear approximation function $L(x,y,z)$ near the point P.
d) Use it to evaluate the linear approximation to $F(0.99, 1.02, -1.03)$.

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: