MAT2705-01/02 13F Quiz 10 NOT! Print Name (Last, First) ____________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

a) Rewrite this matrix system of DEs **and** its initial conditions explicitly in scalar form as a sequence of equations separated by commas.

b) Maple solves a set of scalar differential equations, not a matrix differential equation, so write this same sequence of equations in the input region in Maple and right click to solve the system, but using function notation for the unknowns (i.e., $x1(t), x1'(t)$), no need to make subscripts. Write down the Maple solution.

c) Plot for one period of the frequency $\omega = 2$ oscillations centered about $t = 0$, namely

$> plot\left([\ \ldots, \ldots], t = -\frac{\pi}{2} .. \frac{\pi}{2}, color = [red, blue] \right)$

where you list the two expressions for the two unknowns in the right order.

d) On a single diagram of the sinusoidal function coefficient parameter space, plot the position vectors of the coefficient vectors (c_1, c_2) of the two sinusoidal solution functions, and draw the corresponding reference triangles. Label these triangles with the exact amplitudes A_i and phase shifts δ_i (expressed using the arctan function and appropriate π terms). Use them to re-express the solution functions as phase-shifted cosines.

e) You can check your polar coordinates in the parameter plane for each of these sinusoidal functions which gave you the above amplitudes and phase-shifts by applying the complex number polar form command to the corresponding complex number. For example, if $(c_1, c_2) = (-2, 2)$, then the input

$> \text{polar}(-2 + 2\,I)$

will give you the output: "$\text{polar}\left(2\sqrt{2}, \frac{3}{4}\pi\right)$" with the radial and angular variables for that point in the complex plane (magnitude and principal argument of the complex number, amplitude and phase shift of the corresponding sinusoidal coefficient vector).

f) Evaluate $\delta_2 - \delta_1$ and $\frac{\delta_2 - \delta_1}{2\pi}$ exactly and then numerically. Maple "combine" will simplify their difference exactly to something recognizable (right click on the difference output, select "Combine"). Does this value agree with the obvious angle you see in the diagram, if you did it correctly that is?

g) Knowing that a positive phase shift means the peaks are shifted right on the time axis, while negative phase shift means the peaks are shifted left on the time axis, does your plot of the two solution functions agree with your assignment of phase shifts? Explain. Which curve (red or blue) is to the right (later in time with its peaks)?

▶ solution