MAT2705-01/02 13F Quiz 8 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (**but give decimal approximations for interpretation**). Indicate where technology is used and what type (Maple, GC).

1. $my''+cy'+ky=0$, $y(0)=-5$, $y'(0)=20$; $m=1/2$, $c=2$, $k=52$. [prime is d/dt]

a) State Maple's solution of the initial value problem.

b) Put the DE into standard linear form first. Then identify the values of the damping constant and characteristic time $k_0=1/\tau_0$, the natural frequency ω_0, and the quality factor $Q=\omega_0\tau_0$, exactly and numerically. Is this underdamped, critically damped or overdamped?

c) Find the general solution by hand, showing all steps.

d) Find the solution satisfying the initial conditions, showing all steps.

e) Give exact and numerical values of the amplitude and phase shift and re-express the sinusoidal factor of this solution in phase-shifted cosine form. [Make sure you use a diagram to justify your values.] State what numerical fraction of a cycle (2π) the phase shift is (i.e., evaluate $\delta/2\pi$), as well as its numerical value in degrees, and whether the cosine curve is shifted left (earlier in time) or right (later in time) on the time line. Explain.

f) State the two envelope functions of this decaying oscillating solution.

g) Make a rough sketch of the plot of your solution and its two envelope functions in a viewing window of width 5 times the characteristic time of the solution exponential factor.

h) What are the numerical values of the periods associated with the natural frequency and the actual frequency of the sinusoidal factor of the solution and how do they compare?

▶ **solution**

1 a) $> \frac{1}{2}y''+2y'+52y=0,\ y(0)=-5,\ y'(0)=20$

$$\boxed{y=e^{-2t}\sin(10t)-5e^{-2t}\cos(10t)}$$

b) Divide by $\frac{1}{2}$ →

$$\boxed{y''+4y'+104y=0}$$

$$\boxed{k_0=4,\ \tau_0=\tfrac{1}{4}} \qquad \omega_0^2=104,\ \boxed{\omega_0=\sqrt{104}\approx 10.1}$$

$$Q=\omega_0\tau_0=\boxed{\tfrac{1}{4}\sqrt{104}\approx 2.55} \gg \tfrac{1}{2}$$

$\boxed{\text{underdamped}}$

c) $y=e^{rt} \rightarrow (r^2+4r+104)e^{rt}=0$

$r^2+4r+104=0$

$$r=\frac{-4\pm\sqrt{16-416}}{2}=-2\pm\frac{\sqrt{400}}{4}i=-2\pm 10i$$

$$e^{rt}=e^{(-2\pm 10i)t}=e^{-2t}e^{\pm 10it}$$

$$=e^{-2t}(\cos 10t\pm i\sin 10t)$$

$\hookrightarrow e^{-2t}\cos 10t,\ e^{-2t}\sin 10t$ (real soln basis)

$$\boxed{y=c_1e^{-2t}\cos 10t+c_2e^{-2t}\sin 10t = e^{-2t}(c_1\cos 10t+c_2\sin 10t)}$$

d) $y'=-2e^{-2t}(c_1\cos 10t+c_2\sin 10t)$
$\quad +e^{-2t}(-10c_1\sin 10t+10c_2\cos 10t)$

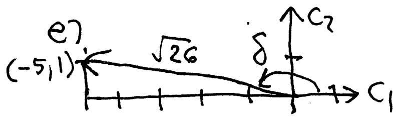
d) $y(0)=c_1=-5$

$y'(0)=-2c_1+10c_2=20$

$$c_2=\frac{20+2(-5)}{10}=1$$

$$\boxed{y=e^{-2t}(-5\cos 10t+\sin 10t)}$$

e)



$$\boxed{\begin{aligned}\delta&=\pi-\arctan(1/5)\\&\approx 2.944\text{ radians}\\&\approx 168.7^\circ\end{aligned}}$$

$$A=\sqrt{(-5)^2+(1)^2}=\boxed{\sqrt{26}}$$

$$\boxed{y=\sqrt{26}\,e^{-2t}\cos(10t-(\pi-\arctan 1/5))}$$

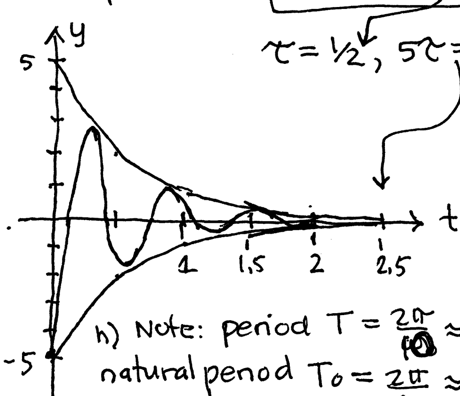
$$\boxed{\delta/2\pi\approx 0.47\text{ cycles}} \qquad \boxed{A\approx 5.10}$$

This is almost a half cycle to the right on the time line, so lagging behind in time compared to the standard cosine cycle.

f) envelope curves: $\boxed{\pm\sqrt{26}\,e^{-2t}\approx\pm 5.10e^{-2t}}$

$\tau=1/2,\ 5\tau=2.5$

g)



h) Note: period $T=\frac{2\pi}{10}\approx 0.628$

natural period $T_0=\frac{2\pi}{\omega_0}\approx 0.616$, a bit smaller