MAT2705-01/02 13F Quiz 7 Print Name (Last, First) ____________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $y'' + 8y' + 15y = 0,\ y(0) = 1,\ y'(0) = 1$ .

a) Find the general solution $y(x)$ of the DE.

b) Using 2x2 matrix methods find the solution which satisfies the initial conditions, showing all work.

c) Use technology to plot your result for $x = 0..1$ and make a rough sketch of what you see, labeling the axes with variable names and key tickmarks on your sketch. Locate on your sketch and label the maximum point found in part d) with its pair of coordinates approximated to 3 decimal places. (Clicking on the gridlines icon for your Maple plot helps you make your sketch more accurate.)

d) Use calculus to determine ***exactly*** by hand (rules of exponents and logs! simplify your result for $y$ to a simple term) the $x$ and $y$ values of the obvious maximum point on the graph and then their approximate values to 3 decimal places, but if you get stuck on solving the derivative condition exactly, use technology to find the approximate values to 4 decimal places in any way you can. Do the numbers you found agree with what your eyes see in the technology plot? [Yes or no, with an explanation would be a good response.]

e) What are the two characteristic lengths $\tau_1 < \tau_2$ for the two decaying exponentials in this problem? What is the value of $5\,\tau_2$? Compare with the choice of horizontal window $x = 0..1$.

## ▶ solution

a) $15[y = e^{rx}]$
$8[y' = re^{rx}]$
$1[y'' = r^2e^{rx}]$
$+$ ________________
$y'' + 8y' + 15y = (r^2 + 8r + 15)e^{rx} = 0$

$r^2 + 8r + 15 = 0$

$r = \ldots = -3, -5$

maple
quad formula
whatever

$e^{rx} = e^{-3x}, e^{-5x}$ basis

$\boxed{y = c_1 e^{-3x} + c_2 e^{-5x}}$ gen Soln

b) $y' = -3c_1 e^{-3x} - 5c_2 e^{-5x}$

$y(0) = c_1 + c_2 = 1$

$y'(0) = -3c_1 - 5c_2 = 1$

$$\begin{bmatrix} 1 & 1 \\ -3 & -5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} -5 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -6/-2 \\ 4/-2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$\boxed{y = 3e^{-3x} - 2e^{-5x}}$

d) $y' = -9e^{-3x} + 10e^{-5x} = 0$

$\frac{10}{9} = e^{(5-3)x} = e^{2x} \rightarrow \boxed{x = \frac{1}{2}\ln\frac{10}{9}} = \ln\left(\frac{10}{9}\right)^{1/2} = \ln\frac{\sqrt{10}}{3}$

$\boxed{= -\frac{1}{2}\ln\frac{9}{10}}$

d) continued

$y = 3e^{-\frac{3}{2}\ln\frac{10}{9}} - 2e^{-5/2\ln\frac{10}{9}}$

$= 3\left(e^{\ln\frac{10}{9}}\right)^{-\frac{3}{2}} - 2\left(e^{\ln\frac{10}{9}}\right)^{-\frac{5}{2}}$

$= 3\left(\frac{10}{9}\right)^{-3/2} - 2\left(\frac{10}{9}\right)^{-5/2}$

$= 3\left(\frac{9}{10}\right)^{3/2} - 2\left(\frac{9}{10}\right)^{5/2}$

$= 3\left(\frac{9}{10}\right)^{3/2} - 2\cdot\frac{9}{10}\left(\frac{9}{10}\right)^{3/2}$

$= \left(3 - \frac{18}{10}\right)\left(\frac{9}{10}\right)^{3/2} = \frac{6}{5}\frac{(9^{1/2})^3}{10^{3/2}}$

$\frac{12}{10} = \frac{6}{5}$ tedious yes I know $= \frac{6 \cdot 27}{5 \cdot 10^{3/2}} = \boxed{\frac{81}{250}\sqrt{10}}$ !

Maple agrees

3 decimal places

$\boxed{\begin{matrix} x \approx 0.053 \\ y \approx 1.025 \end{matrix}}$

e) $y = C_1 e^{-3x} + C_2 e^{-5x}$

$\tau = \frac{1}{3} > \tau = 1/5$

$\boxed{\tau_1 = \frac{1}{5},\ \tau_2 = \frac{1}{3}}$

$\boxed{5\tau_2 = 5/3}$ bigger than $x = 0..1$

note: basic units are always positive! they set the scale for a quantity have you ever heard of a negative foot or second? no, you can subtract these, but "minus 2 feet"? nope!!

c)