MAT2705-01/02 13F Quiz 6 Print Name (Last, First) ______________________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **[You should use technology for row reductions and determinants. Report results you find in this way: value of det, reduced matrix.]**

1. a) Find the general solution of the following linear system, identifying your coefficient matrix A, the rhs vector $\vec{b}$, the augmented matrix, the rref matrix and the vector form $\vec{x} = \ldots$ of the solution.

$$\begin{aligned} x_1 + 2\,x_2 + 3\,x_3 + 4\,x_5 &= -1 \\ 2\,x_1 + 3\,x_2 + 5\,x_3 + x_4 + 7\,x_5 &= 0 \\ 3\,x_1 + x_2 + 5\,x_3 + 5\,x_4 + 7\,x_5 &= 7 \end{aligned}$$

b) Identify a basis of the solution space of the related homogeneous system corresponding to setting $\vec{b} = \vec{0}$.

c) Consider the set of vectors $\{\vec{v}_1, \ldots, \vec{v}_5\}$ which form the columns of A. What are the independent relationships among this vectors? (Write each in the form: a linear combination of them equals the zero vector.)

d) What subset of these vectors does our solution algorithm show to be linearly independent automatically? Express $\vec{b}$ as a unique linear combination of these latter vectors.

2. Which of the following sets of vectors are linearly independent? Justify your claim. Report the results of any calculation you use in doing so.

a) $\{\langle 3,-2,1\rangle \mid \langle 6,1,0\rangle \mid \langle 0,-5,2\rangle\}$

b) $\{\langle 2,1,-3\rangle \mid \langle 4,0,1\rangle \mid \langle 7,1,-1\rangle\}$

c) $\{\langle 3,-2,1,1\rangle \mid \langle 6,1,0,2\rangle \mid \langle 0,-5,2,3\rangle\}$ [Hint: Cursor in any last row entry, "Control Shift R" will add a column to an existing matrix.]

▶ solution

$$\langle A|\vec{b}\rangle = \begin{bmatrix} 1 & 2 & 3 & 0 & 4 & -1 \\ 2 & 3 & 5 & 1 & 7 & 0 \\ 3 & 1 & 5 & 5 & 7 & 7 \end{bmatrix} \xrightarrow[\text{Maple}]{\text{rref}} \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 3 \\ 0 & 1 & 0 & -1 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ L & L & L & F & F \end{matrix} \end{array}$$

1 a)
$$\underbrace{\begin{bmatrix} 1 & 2 & 3 & 0 & 4 \\ 2 & 3 & 5 & 1 & 7 \\ 3 & 1 & 5 & 5 & 7 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -1 \\ 0 \\ 7 \end{bmatrix}}_{\vec{b}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 2 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_1 + 2x_4 + 2x_5 &= 3 \\ x_2 - x_4 + x_5 &= -2 \\ x_3 &= 0 \end{aligned} \quad \xrightarrow{\text{backsub}} \quad \begin{aligned} x_4 &= t_1 \\ x_5 &= t_2 \\ &(\text{free}) \end{aligned} \quad \begin{aligned} x_1 &= 3 - 2t_1 - 2t_2 \\ x_2 &= -2 + t_1 - t_2 \\ x_3 &= 0 \\ x_4 &= t_1 \\ x_5 &= t_2 \end{aligned}$$

$$\boxed{\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3-2t_1-2t_2 \\ -2+t_1-t_2 \\ 0 \\ t_1 \\ t_2 \end{bmatrix}} = \underbrace{\begin{bmatrix} 3 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\vec{x}_p} + \underbrace{t_1 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -2 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{\vec{x}_h}$$

b) $\boxed{\{\langle -2,1,0,1,0\rangle, \langle -2,-1,0,0,1\rangle\}}$ is a basis of the hom. soln space

c) These are the coefficient vectors of the linear relationships
$$\boxed{\begin{aligned} -2\vec{v}_1 + \vec{v}_2 + \vec{v}_4 &= \vec{0} \\ -2\vec{v}_1 - \vec{v}_2 + \vec{v}_5 &= \vec{0} \end{aligned}}$$

d) Leading columns are lin. ind. subset: $\boxed{\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}}$

and $\vec{x}_p$ is the unique coefficient vector for this subset: $\boxed{\langle -1, 0, 7\rangle = 3\vec{v}_1 - 2\vec{v}_2 + 0\vec{v}_3}$

2 a) $\begin{vmatrix} 3 & 6 & 0 \\ -2 & 1 & -5 \\ 1 & 0 & 2 \end{vmatrix} = 0$ so $\boxed{\text{lin. dep.}}$

b) $\begin{vmatrix} 2 & 4 & 7 \\ 1 & 0 & 1 \\ -3 & 1 & -1 \end{vmatrix} = -3 \neq 0$ so $\boxed{\text{lin. ind.}}$

c) $\begin{bmatrix} 3 & 6 & 0 \\ -2 & 1 & -5 \\ 1 & 0 & 2 \\ 1 & 2 & 3 \end{bmatrix} \xrightarrow{\text{rref}} \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} L & L & L \end{matrix} \end{array}$

no free columns so $\boxed{\text{lin. ind.}}$ the associated homogeneous linear system has only the $\vec{0}$ soln for coefficients x_1, x_2, x_3 of the 3 columns.