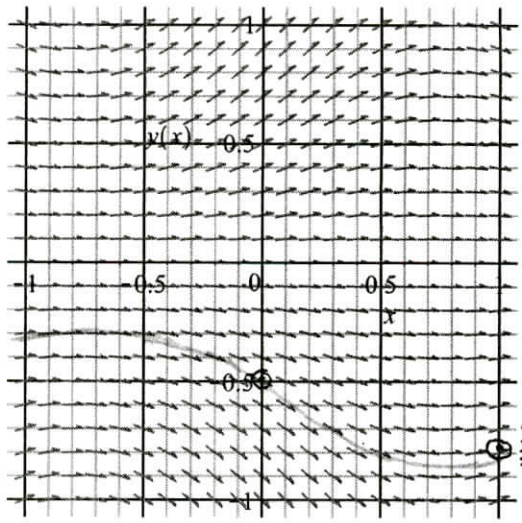
MAT2705-01/02 13F Quiz 2 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\frac{dy}{dx} = y\cos(2x), y(0) = -\frac{1}{2}$.

a) Indicate the initial data point on the graph by a circled dot and roughly draw in the corresponding solution curve.
b) Find the general solution of this separable differential equation, making sure you redefine your final constant to include the obvious missing solution from your family.
c) Find the solution which satisfies the initial condition (no decimal point numbers!).
d) Evaluate $y(1)$ for this solution and mark the corresponding point on the graph by a circled dot. Is this consistent with your approximate hand drawn solution? Explain.



e) Check by hand that your solution to c) solves the differential equation. [Remember, backsub everywhere in the DE eliminating y, then simplify both sides independently.]
f) Enter the differential equation and the initial condition separated by a comma in Maple. Right click and solve. Write down the form of the solution that it gives you. Does it agree with your hand solution?

▶ solution

distinguish different constants in same calculation

a) see graph

b) $\frac{dy}{dx} = y\cos 2x$

$\int \frac{dy}{y} = \int \cos 2x\,dx$ ↓"short list"

$\ln|y| = \frac{1}{2}\sin 2x + C_1$

$e^{(\ln|y|)} = e^{C_1} e^{\frac{1}{2}\sin 2x}$

$|y| = e^{C_1} e^{\frac{1}{2}\sin 2x}$

$y = \pm e^{C_1} e^{\frac{1}{2}\sin 2x}$

C

$\boxed{y = Ce^{\frac{1}{2}\sin 2x}}$

always let final simplified constant be C with no subscript.

c) $y(0) = -\frac{1}{2} \rightarrow x = 0, y = -\frac{1}{2}$

$-\frac{1}{2} = Ce^{\frac{1}{2}\sin(0)} = C$ ← not soln

$e^0 = 1$

$\boxed{y = -\frac{1}{2}e^{\frac{1}{2}\sin 2x}}$ ← this is the soln

d) $y(1) = -\frac{1}{2}e^{\frac{1}{2}\sin 2}$ ← 2 radians not 2 degrees!

$\approx \boxed{-0.78780}$

my hand drawn curve is a bit low but given the poor resolution of the grid arrows, it seems reasonable.

e) $y = -\frac{1}{2}e^{\frac{1}{2}\sin 2x}$

$\frac{dy}{dx} = -\frac{1}{2}e^{\frac{1}{2}\sin 2x}\left(\frac{1}{2}(\cos 2x)(2)\right) = -\frac{1}{2}\cos 2x\, e^{\frac{1}{2}\sin 2x}$

$\frac{dy}{dx} = y\cos 2x \rightarrow -\frac{1}{2}\cos 2x\, e^{\frac{1}{2}\sin 2x} = \left(-\frac{1}{2}e^{\frac{1}{2}\sin 2x}\right)\cos 2x$ ✓

$= -\frac{1}{2}\cos 2x\, e^{\frac{1}{2}\sin 2x}$ ✓(same)

eliminate y everywhere, equation must be true for all x.

f) $\boxed{y(x) = -\frac{1}{2}e^{\frac{1}{2}\sin(2x)}}$

Same except I did not include parentheses—standard math notation