MAT2705-01/02 13F Test 1 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). [Recall you need $y'(t), y(t)$ instead of y', y in your differential equation for an unknown variable y for Maple to interpret the prime as a t derivative.]

1. $\frac{dP}{dt} = 3\,e^{-5t} P$, $P(0) = 100$ [combine any exponentials that occur in this problem!]

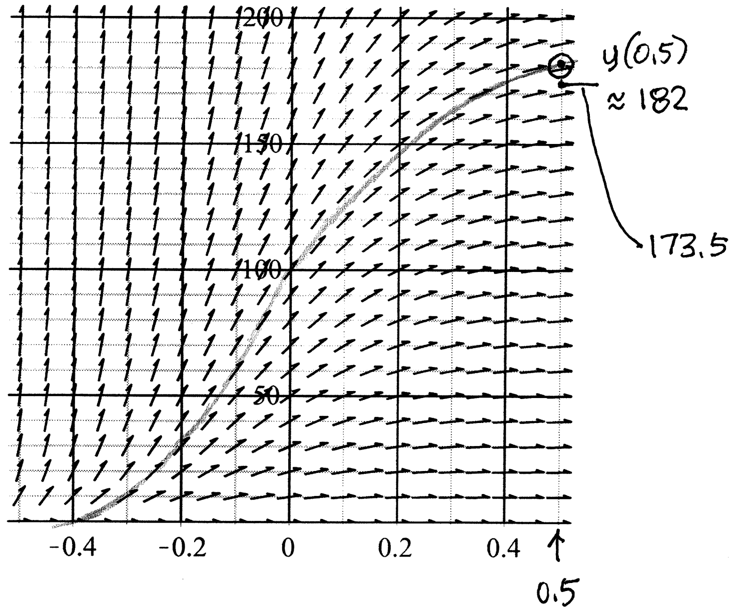
a) Find the general solution using the separable technique.

b) Find the solution satisfying the stated initial condition and evaluate it at $t = 0.5$ to 4 decimal place accuracy.

c) Draw the corresponding solution curve on the slope field for all values of t in the plot window and estimate the value of P at $t = 0.5$. Compare your graphical and numerical values.

d) What is the limiting value of P for your IVP solution as $t \to \infty$?

e) What is the value of the characteristic time τ for the exponential function in this differential equation? Make a rough sketch of your choice of technology plot of your IVP solution from $t = 0$ to $t = 5\,\tau$, including the horizontal asymptote corresponding to part d) as well as the time axis.



f) Show by hand that your IVP solution satisfies the differential equation.

g) Does your IVP solution agree with Maple's exact result? If not show how they can be made to agree.

2. $\frac{dP}{dt} = 3\,e^{-5t} P + Q_0$, Q_0 a constant.

a) Put this equation into the standard form for a first order linear differential equation.

b) Evaluate the integrating factor.

c) Proceed to solve the differential equation to state the general solution for $P(t)$, leaving your indefinite integral unevaluated (it requires a special function!).

d) If you set $Q_0 = 0$, does it reduce to your result in Problem 1?

e) **Optional.** Use Maple to solve this differential equation with $Q_0 = 1$ and write down the solution.

Be sure to sign and date the pledge before handing in this test.

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

MAT2705-01/02 13F Test 1 Answers

① a) $\frac{dP}{dt} = 3e^{-5t}P$

$\int \frac{dP}{P} = \int 3e^{-5t}\,dt$

$\ln|P| = \frac{3e^{-5t}}{-5} + C_1$

$|P| = e^{\ln|P|} = e^{\left(-\frac{3}{5}e^{-5t} + C_1\right)}$

$= e^{C_1}e^{-3/5 e^{-5t}}$

$P = \underbrace{\pm e^{C_1}}_{C} e^{-3/5 e^{-5t}}$

$\boxed{P = C e^{-\frac{3}{5}e^{-5t}}}$

b) $100 = P(0) = C e^{-\frac{3}{5}e^{0}}$

$= Ce^{-3/5} \rightarrow C = 100e^{3/5}$

$P = 100e^{3/5}e^{-3/5 e^{-5t}}$

$= 100e^{3/5 - 3/5 e^{-5t}}$ combine exp's

$= 100e^{\frac{3}{5}(1-e^{-5t})}$ and factor!

$\boxed{P = 100e^{3/5(1-e^{-5t})}}$

$P(0.5) = 100\, e^{3/5(1-e^{-2.5})}$

≈ 173.455172

$\approx \boxed{173.4552}$ ~~4-dec~~ 4-place accuracy

c) My graphical value is a little high at 182, but this is only about 5% off. I am actually a bit surprised but graphical work is pretty rough,

d) $\lim_{t\to\infty} P = \lim_{t\to\infty} 100e^{3/5(1-e^{-5t})}$, $e^{-5t} \to 0$

$= \boxed{100\,e^{3/5}} \approx 182.211880$

$\approx \boxed{182.2}$

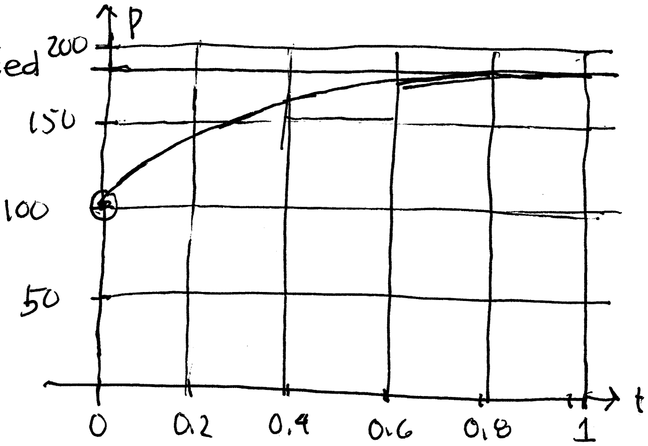
e) easy: $e^{-5t} \rightarrow \boxed{\tau = 1/5 = 0.2}$

$5\tau = 5/5 = 1$ so $t = 0..1$

H Asymp: $P = P_\infty \approx 182.2$

show $P = 0..200$.

e) continued



rough hand sketch.

f) $P = 100\, e^{3/5(1-e^{-5t})}$

$\frac{dP}{dt} = 100\, e^{3/5(1-e^{-5t})}\left(\frac{3}{5}(0 - e^{-5t}(-5))\right)$

$= 100\, e^{3/5(1-e^{-5t})}\, 3e^{-5t}$

$\frac{dP}{dt} = 3e^{-5t}P$

$100e^{3/5(1-e^{-5t})}(3e^{-5t}) \overset{\checkmark}{=} 3e^{-5t}\,100e^{3/5(1-e^{-5t})}$

(multiplication is commutative)

$300e^{-5t}e^{3/5(1-e^{-5t})} = 300e^{-5t}e^{3/5(1-e^{-5t})}$ ✓

g) Maple: $P(t) = \frac{100e^{-3/5e^{-5t}}}{e^{-3/5}} = 100\,e^{\frac{3}{5}-\frac{3}{5}e^{-5t}}$ agreement

combine

② a) $\frac{dP}{dt} = 3e^{-5t}P + Q_0$

$e^{\frac{3}{5}e^{-5t}}\left[\frac{dP}{dt} - 3e^{-5t}P = Q_0\right]$

b) $e^{\int -3e^{-5t}dt} = e^{\frac{-3}{-5}e^{-5t}} = e^{\frac{3}{5}e^{-5t}}$

c) $\frac{d}{dt}\left(Pe^{\frac{3}{5}e^{-5t}}\right) = Q_0 e^{3/5 e^{-5t}}$

$Pe^{(3/5 e^{-5t})} = \int Q_0 e^{3/5 e^{-5t}}\,dt + C$

$\boxed{P = e^{-\frac{3}{5}e^{-5t}}\left(\int Q_0 e^{3/5 e^{-5t}}\,dt + C\right)}$

d) $Q_0 = 0 \rightarrow P = Ce^{-3/5 e^{-5t}}$ ✓ agreement.

e) $\boxed{P(t) = \left(\frac{1}{5}\mathrm{Ei}\left(1, -\frac{3}{5}e^{-5t}\right) + \_C1\right)e^{-\frac{3}{5}e^{-5t}}}$

"Exponential Integral"