MAT2500-01/04 12S Quiz 5 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. Find $\frac{\partial z}{\partial x}$ at the point $(x, y, z) = (2, -1, 0)$ for the surface $\sin(x y z) = x + 2y + 3z$.

2. a) Evaluate the linear approximation $L(x, y)$ to the function $f(x, y) = x^2 - y^2$ at the point $(2, 1)$ and use it to approximate $f(2.01, 0.98)$.
b) Write the equation for the tangent plane $z = L(x, y)$ and simplify it to the standard form $a x + b y + c z = d$.

3. The lateral surface area of a cone of radius r and height h is $S = \pi r \sqrt{r^2 + h^2}$. For a cone of radius 3 and height 4, use the differential approximation to estimate the increase in surface area if both dimensions increase by 1 percent. First state the differential dS for $(r, h) = (3, 4)$. What is the percentage change in S?

▶ solution

① $\frac{\partial}{\partial x}\left(\sin(xyz) = x + 2y + 3z\right)$

$\underbrace{\cos(xyz)\frac{\partial}{\partial x}(xyz)}_{y\frac{\partial}{\partial x}(xz) \;=\; y(1z + x\frac{\partial z}{\partial x})} = \frac{\partial}{\partial x}(x+2y+3z) = 1 + 0 + 3\frac{\partial z}{\partial x}$

$\cos(xyz)(y)(z + x\frac{\partial z}{\partial x}) = 3\frac{\partial z}{\partial x} + 1$

$yz\cos xyz + xy(\cos xyz)\frac{\partial z}{\partial x} = 3\frac{\partial z}{\partial x} + 1$

$-1 + yz\cos xyz = (3 - xy\cos xyz)\frac{\partial z}{\partial x}$

$\frac{\partial z}{\partial x} = \frac{yz\cos xyz - 1}{3 - xy\cos xyz}$

$\left.\frac{\partial z}{\partial x}\right|_{(x,y,z)=(2,-1,0)} = \frac{0-1}{3-(2)(-1)\cos 0} = \frac{-1}{3+2} = \boxed{-\frac{1}{5}}$

② a) $f(x,y) = x^2 - y^2$ $\quad f(2,1) = 4 - 1 = 3$

$f_x(x,y) = 2x$ $\quad f_x(2,1) = 2(2) = 4$

$f_y(x,y) = -2y$ $\quad f_y(2,1) = -2(1) = -2$

$\boxed{L(x,y) = f(2,1) + f_x(2,1)(x-2) + f_y(2,1)(y-1) = 3 + 4(x-2) - 2(y-1)}$

$= 3 + 4x - 8 - 2y + 2 = 4x - 2y - 3$

b) $z = 4x - 2y - 3$

$\boxed{4x - 2y - z = 3}$

$L(2.01, 0.98) = 3 + 4(2.01-2) - 2(0.98-1)$
$= 3 + 4(.01) + 2(0.02) = \boxed{3.08}$

③ $S = \pi r (r^2+h^2)^{1/2}$

$\frac{\partial S}{\partial r} = \pi(1)(r^2+h^2)^{1/2} + \pi r \frac{1}{2}\frac{2r}{(r^2+h^2)^{1/2}}$
$= \pi(r^2+h^2)^{1/2} + \frac{\pi r^2}{(r^2+h^2)^{1/2}}$

$\frac{\partial S}{\partial h} = \pi r(\tfrac{1}{2})\frac{2h}{(r^2+h^2)^{1/2}} = \frac{\pi r h}{(r^2+h^2)^{1/2}}$

$\left.\frac{\partial S}{\partial r}\right|_{(r,h)=(3,4)} = \pi(5) + \frac{\pi(3)^2}{5} = \frac{34\pi}{5}$

$\left.\frac{\partial S}{\partial h}\right|_{(r,h)=(3,4)} = \frac{\pi(3)(4)}{5} = \frac{12\pi}{5}$

$\boxed{dS|_{(r,h)=(3,4)} = \frac{34\pi}{5}dr + \frac{12\pi}{5}dh}$

$\frac{dr}{r} = .01 \to dr = .01r \xrightarrow{r=3} .03$

$\frac{dh}{h} = .01 \to dh = .01h \xrightarrow{h=4} .04$

$dS = \frac{34\pi(.03) + 12\pi(.04)}{5}$
$= \frac{\pi(1.02 + .48)}{5} = \frac{\pi(1.5)}{5}$
$= 0.3\pi \approx 0.94248$
$\approx \boxed{0.94}$

$\frac{dS}{S} = \frac{0.3\pi}{\pi(3)(5)} = .02 \to \boxed{2\%}$ increase