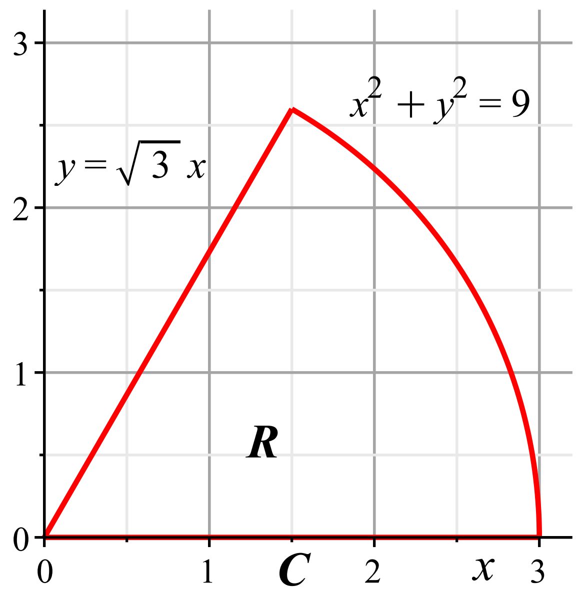
MAT2500-01/04 12S Final Exam   Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Use technology to check any integrals you set up.

1. Given the point $(x, y, z) = (1, -2, -2)$, find the new coordinates, in each case stating the angles both in radians (exactly, using inverse trig functions) and in degrees (1 decimal place accuracy) and use proper identifying symbols for all coordinates: a) cylindrical coordinates. b) spherical coordinates. Support your work with two diagrams, one of the x-y plane and one of the r-z half plane, each including a reference triangle locating the point with respect to the axes with all three sides labeled by their lengths and both axes labeled by their coordinate labels and showing the relevant angles. Show clearly how you obtain values of your coordinates from these diagrams. Do the angles look right in these diagrams?



2.a) Describe the region $\boldsymbol{R}$ by giving the appropriate intervals of the polar coordinates over the region, and draw in the diagram a typical radial cross-section, labeling its endpoints by the values of the radial coordinate, shading in the region with equally spaced radial cross-sections. State the ranges of the two polar coordinates as inequalities.

b) Use polar coordinates to evaluate the three integrals $A = \iint_{\boldsymbol{R}} 1\, dA$, $A_x = \iint_{\boldsymbol{R}} x\, dA$ and $A_y = \iint_{\boldsymbol{R}} y\, dA$ by hand exactly. Evaluate the coordinates $\overline{x} = A_x / A$, $\overline{y} = A_y / A$ of the centroid of the region R exactly and numerically. Locate the centroid on the diagram and identify it. Does it seem right? Explain.

c) Now represent A_x by a single iterated integral in Cartesian coordinates and evaluate it exactly by hand or with technology as a partial check on your work.

d) Evaluate the counterclockwise line integral of the vector field $\vec{F}(x, y) = \langle x, x\,y \rangle$ around the boundary curve of R.

e) Use Green's theorem $\int_C \vec{F} \cdot d\vec{r} = \iint_{\boldsymbol{R}} \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dA$ to evaluate the same line integral.

You should get the same result.

f) Given the angle subtended by this sector, using elementary geometry, state the lengths of the 3 components of the bounding curve C around the sector and their sum S, which is the value of the total arclength integral $S = \int_C ds$.

g) Evaluate $C_y = \int_C y\, ds$ over the boundary of this region. Then by symmetry (optional: why?),

$\langle \overline{x}, \overline{y} \rangle = \frac{\langle \sqrt{3}\, C_y, C_y \rangle}{S}$ is the centroid of the bounding curve C. Locate it on the diagram and identify it.

How does this compare to the centroid of the sector itself?

3. $\vec{F} = \langle (1 + x\,y)\, e^{xy}, e^y + x^2\, e^{xy} \rangle$

a) Set up a definite integral that represents the line integral of $\vec{F}$ on the line segment from $(1, 0)$ to $(0, 1)$. Use technology to evaluate it exactly. [You must use the palette for the exponential, and put a space between x and y and any products.]

b) Show that $\vec{F}$ satisfies the condition that it admit a potential function, i.e., is a conservative vector field.

c) Find a potential function f for it.

d) Use the potential to evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ over any curve from $(1, 0)$ to $(0, 1)$. Does it agree with part a) as it should?

▶ solution (on-line)

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in with your answer sheets as a cover page, with the Lastname, FirstName side face up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: