MAT2500-01/04 12S Takehome Test 3 Print Name (Last, First) _____|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Explain in as many words as possible everything you are doing! For each hand integration step, state the antiderivative formula used before substituting limits into it:

$$\int_a^b f(x)\ \mathrm{d}x = F(x)\big|_{x=a}^{x=b} = F(b) - F(a)\ .$$

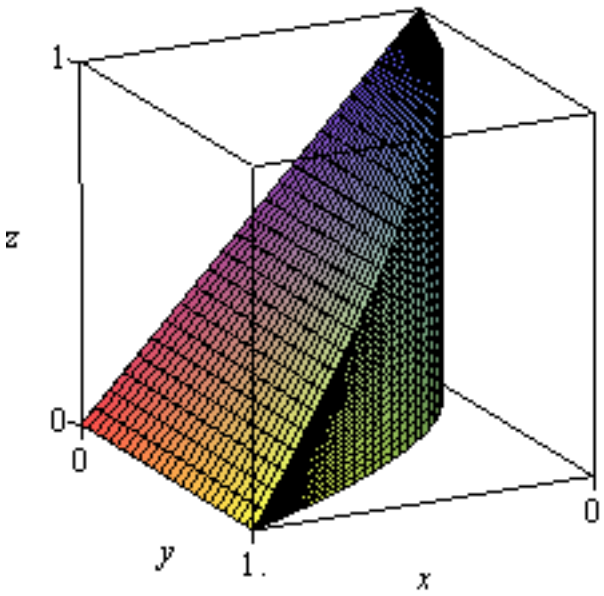
1. A "gross exponential": $G = \int_0^4 \int_{3y}^{12} 6\, \mathrm{e}^{x^2}\, \mathrm{d}x\, \mathrm{d}y$ a) Evaluate this exactly using technology and then numerically evaluate the result.
b) Make a completely labeled (shaded) diagram of the region of integration for this integral, with a typical labeled cross-section linhe segment (bullet endpoints) representing the current iteration of the integral.
c) Make a new completely labeled diagram corresponding to the reversed order of integration.
d) State the new integral with the order of integration reversed.
e) Evaluate the new integral step by step exactly by hand and using technology.
f) Do you get the same result as in part a)? If not find your error.
g) Do you understand the initial pun?

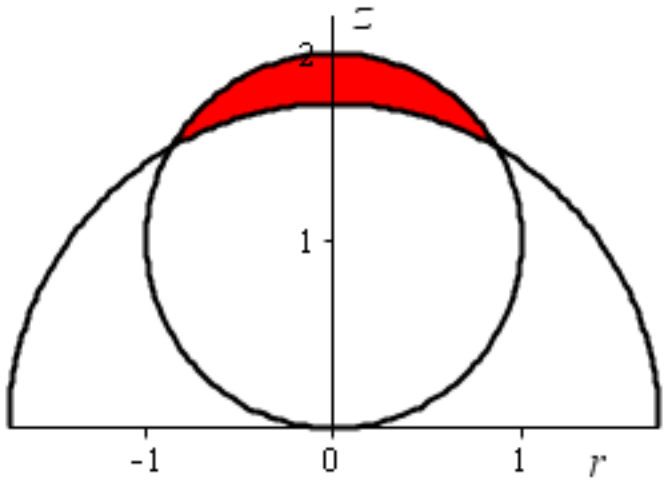
2. Consider the solid region *R* in the first octant corresponding to the region of integration in the triple integral $\int_0^1 \int_0^{\sqrt{x}} \int_0^{1-x} f(x, y, z)\ \mathrm{d}z\, \mathrm{d}y\, \mathrm{d}x$. See the figure on page 2 (left).
a) Make labeled plane diagrams of the projection of *R* onto the *x*-*y* plane (with a labeled cross-section for the inner integration of the outer double integral) and the *x*-*z* plane (with a labeled cross-section for the innermost integral), in each case labeling the cross-section by the starting and stopping value equations for the variable of integration.
b) Rewrite the integral in the order $\iiint$ ... *dx dz dy*, supporting your outer limits of integration with a new labeled diagram as in part a).
c) Rewrite the integral in the order $\iiint$ ... *dy dz dx*, supporting your limits of integration with 2 labeled diagrams as in part a).
d) Evaluate all 3 integrals exactly by hand step by step for $f(x, y, z) = 1$ to get the volume of this region. Your results should agree.

3. Consider the solid of revolution *R* outside the sphere $x^2 + y^2 + z^2 = 3$ and inside the sphere $x^2 + y^2 + (z-1)^2 = 1$ whose vertical cross-section is given in the figure (next page, but note $r \geq 0$, while the diagram shows this cross-section revolved around the vertical axis as well).
a) Express the equation for these two surfaces first in cylindrical coordinates and then in spherical coordinates, simplifying both. For cylindrical coordinates solve for the vertical variable in terms of the radial variable, and for spherical coordinates, express the radial variable as a function of the angular variable in the *r*-*z* half plane. What is the smallest value of *z* that occurs in this solid? The largest value of *r*?
b) Make a new *r*-*z* half-plane diagram illustrating a typical cylindrical coordinate vertical cross-section with a superimposed arrow for its direction, labeling its endpoints by the starting and stopping values of the vertical coordinate, and describe the range of values of the remaining coordinates over this region.
c) From your diagram write down an interated triple integral in cylindrical coordinates representing the volume of this solid and evaluate it exactly by hand step by step.
d) Repeat a new diagram for spherical coordinates, showing a typical radial cross-section with a superimposed arrow for its direction and labeling its endpoints by the starting and stopping values of the new radial coordinate and describe the range of values of the remaining coordinates over this region.
e) From your new diagram write down an interated triple integral in spherical coordinates representing the

volume of this solid and evaluate it exactly by hand step by step.
f) Do your two results agree with each other and the Maple evaluation of the triple integrals? If not find your error.
g) The centroid (center of volume = center of mass for a homogeneous mass distribution with density function equal to 1) of this solid lies on the z axis because of the rotational symmetry about this axis. Choose one of these two coordinate systems to evaluate the z coordinate " $\overline{z}$ " of the centroid exactly and numerically.
h) Mark your centroid point on one of your diagrams, identifying it. Is there more volume above the plane $z = \sqrt{3}$ or below this plane? Where do you expect the centroid to lie then along the axis? Explain. Does your numerical value correspond to this reasoning? Are you surprised by the actual numerical value?

4. a) Use spherical coordinates to represent $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} x\,y\,dz\,dy\,dx$ as a triple integral, supporting your limits of integration with labeled diagrams. as above.
b) Evaluate this new integral exactly with technology and compare its value with the original triple integral in Cartesian coordinates evaluated with technology. Do they agree?
c) Evaluate the spherical coordinate integral exactly by hand step by step. Does it agree with your previous results?





solution (on-line)

No collaboration. You may only talk to bob. See test rules on-line. Read short rules above. Print out and attach any Maple supporting work you do, hand annotating if necessary with problem number and part etc, whatever is necessary for clarification.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in stapled to your answer sheets as the cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: