

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). You are encouraged to use technology to check all of your hand results.

$$1. f(x, y) = \frac{5x}{x^2 + y^2}.$$

- Find a unit vector in the direction in which this function increases most rapidly at $(1, 2)$.
- Evaluate the directional derivative of this function at $(1, 2)$ in the direction parallel to the vector $\langle 2, -1 \rangle$.
- What is the slope of the normal line to the contour passing through $(1, 2)$? What is the equation of this contour in simplest form?
- Find the linear approximation function $L(x, y)$ near the point $(1, 2)$.
- Use it to evaluate the linear approximation to $f(.98, 2.01)$.

$$2. f(x, y) = x^3 - 3x - y^3 + 12y$$

- Verify by substitution that $(1, 2)$, $(-1, 2)$ are two critical points of f for which $y > 0$.
- Use the second derivative test to classify both of these critical points as local minima, local maxima or saddle points. Use words to explain your reasoning.
- Does a quick contourplot of this function confirm your conclusions? Explain. (Right click on function, plots, plot builder, use a small enough window or the option contours = 50).

$$3) \text{ If } yz + x \ln(y) = z^2 + 1, \text{ find } \frac{\partial y}{\partial z} \text{ and evaluate it at the point } (1, e, e).$$

$$4. \text{ a) Find the equation of the tangent plane to the ellipsoid } \frac{x^2}{4} + y^2 + \frac{z^2}{3} = 5 \text{ at the point } (-2, 1, -3), \text{ and}$$

simplify it to the standard linear form. Identify a (simplest) normal vector \vec{n} to this plane.

- What are the parametrized equations for the normal line to this surface at the same point using this normal?
- Backsubstitute your equations for the normal line into the ellipsoid equation to determine the parameter value and coordinates of the *other* point where this line passes through the ellipsoid.

5. **Optional no credit**, perhaps for "fun" after the test: the contours of the function in problem 1 are circles. Find the center and radius of the contour which passes through the point (x_0, y_0) .

► solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: _____

Date: _____