MAT2705-01/02 12F Quiz 8 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (**but give decimal approximations for interpretation**). Indicate where technology is used and what type (Maple, GC).

1. $my'' + cy' + ky = 0$, $y(0) = 0$, $y'(0) = -8$; $m = 2$, $c = 12$, $k = 50$. [prime is d/dt]

a) Put the DE into standard linear form first. Then identify the values of the damping constant $k_0 = 1/\tau_0$, the natural frequency ω_0, and the quality factor $Q = \omega_0 \tau_0$. Is this underdamped, critically damped or overdamped?

b) Find the general solution by hand.

c) Find the solution satisfying the initial conditions.

d) Re-express the sinusoidal factor of this solution in phase-shifted cosine form to obtain the two envelope functions of this decaying oscillation solution. State the two envelope functions, and state what fraction of a cycle (2π) the phase shift is and whether the cosine is shifted left (earlier in time) or right (later in time) on the time line.

d) Make a rough sketch of the plot of your solution and its two envelope functions in a viewing window of width 5 times the characteristic time of the solution exponential factor.

e) State Maple's solution of the initial value problem.

▶ solution

a) $2y'' + 12y' + 50y = 0$

$\boxed{y'' + 6y' + 25y = 0}$ standard form.

$y = e^{rt}:\ \to (r^2 + 6r + 25)e^{rt} = 0$

$r^2 + 6r + 25 = 0$

$r = \dfrac{-6 \pm \sqrt{36 - 4\cdot 25}}{2} = -\dfrac{6 \pm \sqrt{-64}}{2}$

$= -3 \pm \frac{8}{2} i = -3 \pm 4i$

$e^{rt} = e^{-3t} e^{\pm 4it} = e^{-3t}(\cos 4t \pm i \sin 4t)$

↳ real basis linear soln space:

$e^{-3t}\cos 4t,\ e^{-3t}\sin 4t$

gen soln: $\boxed{y = e^{-3t}(c_1 \cos 4t + c_2 \sin 4t)}$

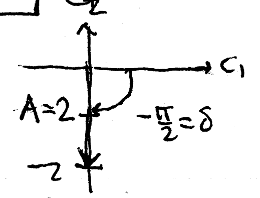
b) $y' = -3e^{-3t}(c_1 \cos 4t + c_2 \sin 4t)$
$\quad + e^{-3t}(-4c_1 \sin 4t + 4c_2 \cos 4t)$

$y(0) = c_1 = 0$

$y'(0) = -3c_1 + 4c_2 = -8 \to c_2 = -2$

$\boxed{y = -2e^{-3t} \sin 4t}$

d) $(c_1, c_2) = (0, -2)$:



$y = 2e^{-3t} \cos(4t - (-\tfrac{\pi}{2}))$

$k = 3,\ \tau = 1/3$

d) continued: $\boxed{y = 2e^{-3t} \cos(4t + \tfrac{\pi}{2})}$

$\omega = 4$

$T = \frac{2\pi}{4} = \frac{\pi}{2} \approx 1.57$

envelopes: $\boxed{y = \pm 2e^{-3t}}$

$\dfrac{\delta}{2\pi} = -\dfrac{\pi/2}{2\pi} = -1/4$ $\boxed{\text{1/4 cycle, to left}}$ on graph ($\delta < 0$) $\boxed{\text{earlier}}$ in time

a) continued:

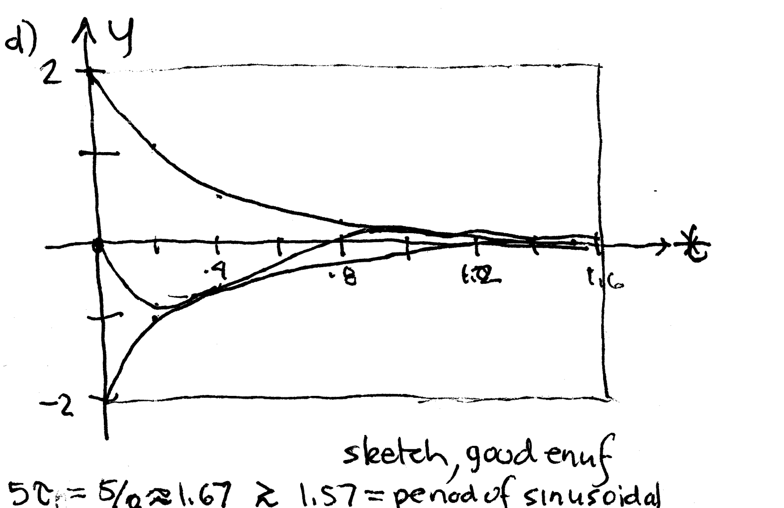
compare $y'' + k_0 y' + \omega_0^2 y = 0$

$\boxed{k_0 = 6,\ \tau_0 = 1/6}$, $\omega_0 = \sqrt{25} = \boxed{5}$,

$Q = \omega_0 \tau_0 = \boxed{\tfrac{5}{6}} > \tfrac{1}{2}$ $\boxed{\text{underdamped}}$

(clear since complex roots!)

d)



sketch, good enuf

$5\tau_0 = 5/3 \approx 1.67 \gtrsim 1.57$ = period of sinusoidal factor

viewing window comparable to period.