MAT2705-01/02 12F Quiz 4 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $3x_1 - x_2 - 3x_3 + 3x_4 = 7$
   $2x_1 - x_2 - 2x_3 + x_4 = 3$
   $x_1 - x_3 + 2x_4 = 4$

a) Write down the coefficient matrix **A**, the RHS matrix $\vec{\mathbf{b}}$ and the augmented matrix $\mathbf{C} = \langle \mathbf{A} \mid \vec{\mathbf{b}} \rangle$ for this linear system of equations.

b) With technology (identify your choice!), reduce this matrix **C** step by step to its ReducedRowEchelonForm avoiding fractions (5 easy steps!), recording the intermediate matrices and row operations for each step (as in $R_1 \leftrightarrow R_2$, $R_3 \to R_3 + 2R_1$, $R_1 \to \frac{1}{2} R_1$).

c) Write out the equations that correspond to the reduced matrix. Identify the leading variables and the free variables and solve. State your solution in the scalar form: $x_1 = \ldots$, $x_2 = \ldots$, etc .

d) Check that your solution is a solution of the original equations.

2. Use technology to solve the system (recall the instruction to identify your choice of technology):
   $x_1 + 2x_2 + x_3 = 4$
   $3x_1 + 8x_2 + 7x_3 = 20$
   $2x_1 + 7x_2 + 9x_3 = 23$

This has an integer solution. Check that it is correct (by backsubstitution).

▶ solution

1 a) $A = \begin{bmatrix} 3 & -1 & -3 & 3 \\ 2 & -1 & -2 & 1 \\ 1 & 0 & -1 & 2 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 7 \\ 3 \\ 4 \end{bmatrix}$ $C = \left[\begin{array}{cccc|c} 3 & -1 & -3 & 3 & 7 \\ 2 & -1 & -2 & 1 & 3 \\ 1 & 0 & -1 & 2 & 4 \end{array}\right]$

b) $\begin{bmatrix} 3 & -1 & -3 & 3 & 7 \\ 2 & -1 & -2 & 1 & 3 \\ 1 & 0 & -1 & 2 & 4 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3}$

$\begin{bmatrix} 1 & 0 & -1 & 2 & 4 \\ 2 & -1 & -2 & 1 & 3 \\ 3 & -1 & -3 & 3 & 7 \end{bmatrix} \xrightarrow{\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1}}$

$\begin{bmatrix} 1 & 0 & -1 & 2 & 4 \\ 0 & -1 & 0 & -3 & -5 \\ 0 & -1 & 0 & -3 & -5 \end{bmatrix} \xrightarrow{R_2 \to -R_2}$

$\begin{bmatrix} 1 & 0 & -1 & 2 & 4 \\ 0 & 1 & 0 & 3 & 5 \\ 0 & -1 & 0 & -3 & -5 \end{bmatrix} \xrightarrow{R_3 \to R_3 + R_2}$

$\begin{bmatrix} 1 & 0 & -1 & 2 & 4 \\ 0 & 1 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to$ $x_1 - x_3 + 2x_4 = 4$, $x_2 + 3x_4 = 5$, $0 = 0$ $\to$ $\boxed{\begin{array}{l} x_1 = 4 + t_1 - 2t_2 \\ x_2 = 5 - 3t_2 \\ x_3 = t_1 \\ x_4 = t_2 \end{array}}$

c) $\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ L & L & F & F \end{array} \longrightarrow x_3 = t_1, x_4 = t_2$

d) ~~continued:~~

$3(4+t_1-2t_2) \overset{?}{=} 7$
$-(5-3t_2)$
$-3(t_1)$
$+3(t_4)$
$12 + 3t_1 - 6t_2$
$-5 \quad + 3t_2$
$-3t_1 + 3t_2$
$7 + 0t_1 + 0t_2$ ✓

$2(4+t_1-2t_2) \overset{?}{=} 3$
$-(5 \quad -3t_2)$
$-2(t_1)$
$+(t_2)$
$8 + 2t_1 - 4t_2$
$-5 \quad + 3t_2$
$-2t_1$
$+ t_2$
$3 + 0t_1 + 0t_2$ ✓

$(4+t_1-2t_2) = 4?$
$-(t_1)$
$+2(t_2)$
$4 + t_1 - 2t_2$
$-t_1 + 2t_2$
$4 + 0t_1 + 0t_2$ ✓

2 $\begin{bmatrix} 1 & 2 & 1 & 4 \\ 3 & 8 & 7 & 20 \\ 2 & 7 & 9 & 23 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ Maple RREF

$\to \boxed{\begin{array}{l} x_1 = 5 \\ x_2 = -2 \\ x_3 = 3 \end{array}}$

$(5) + 2(-2) + (3) \overset{?}{=} 4$
$5 - 4 + 3 = 4$ ✓
$3(5) + 8(-2) + 7(3) \overset{?}{=} 20$
$15 - 16 + 21 = 20$ ✓
$2(5) + 7(-2) + 9(3) \overset{?}{=} 23$
$10 - 14 + 27 = 23$ ✓