MAT2705-01/02 12F Quiz 2    Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\dfrac{dy}{dx} = 3\sqrt{x\,y}$

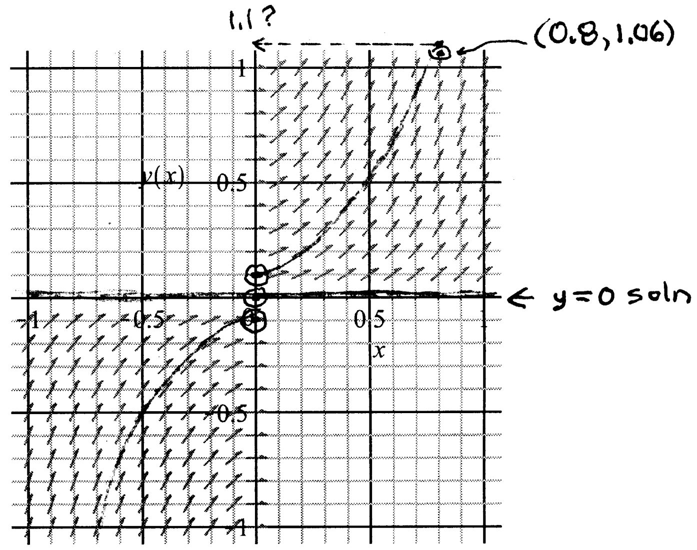
three initial conditions 1: $y(0) = -\dfrac{1}{10}, 0, \dfrac{1}{10}$.

a) Indicate these initial data points on the graph by circled dots and roughly draw in the corresponding solution curves.

b) Find the (almost) general solution of the differential equation. What obvious solution is missing from this family?

c) Find the solution which satisfies the last initial condition (no decimal point numbers!).

d) Evaluate $y(0.8)$ for this solution and mark the corresponding point on the graph by a circled dot. Is this consistent with your approximate hand drawn solution? Explain.




e) Check by hand that your solution to c) solves the differential equation. [Remember, backsub everywhere in the DE eliminating $y$, then simplify both sides independently.]

f) Enter the differential equation and the last initial condition separated by a comma in Maple, BUT express the right hand side in factored form first! Right click and solve. Write down the form of the solution that it gives you. Does it agree with your hand solution? Show that indeed they agree (unless your hand solution is wrong!).

## ▶ solution

1. a) see graph

b) $\dfrac{dy}{dx} = 3\sqrt{xy} = 3x^{1/2}y^{1/2}$ if $x \ge 0, y \ge 0$

$\int y^{-1/2}\,dy = \int 3x^{1/2}\,dx$   separate

$\dfrac{y^{1/2}}{1/2} = 3\dfrac{x^{3/2}}{3/2} + C_1$

$y^{1/2} = x^{3/2} + \tfrac{1}{2}C_1$

$\boxed{y = (x^{3/2} + C)^2}$ (first quadrant)

$y = 0$ is an obvious singular soln. you see it in the direction field plot as an isocline soln!

c) $\frac{1}{10} = (0 + C)^2 = C^2 \rightarrow C = \frac{1}{\sqrt{10}}$

$\boxed{y = (x^{3/2} + 1/\sqrt{10})^2}$

a hair high · not bad!

d) $y(0.8) = ((.8)^{3/2} + \sqrt{10})^2 \approx \boxed{1.06} \Leftrightarrow 1.1$

e) $y = (x^{3/2} + 10^{-1/2})^2$, $y' = 2(x^{3/2} + 10^{-1/2})\tfrac{3}{2}x^{1/2}$

DE: $3(x^{3/2} + 10^{-1/2})x^{1/2} = 3x^{1/2}((x^{3/2} + 10^{-1/2})^2)^{1/2}$
$= 3x^{1/2}(x^{3/2} + 10^{-1/2})$ ✓

f) Maple: $\boxed{y(x) = \underbrace{\tfrac{1}{5}x^{3/2}\sqrt{10}}_{2\frac{1}{\sqrt{10}}x^{3/2}} + x^3 + \tfrac{1}{10}} = (x^{3/2} + \tfrac{1}{\sqrt{10}})^2$

maple rationalizes radicals

$2\,ab + a + b = (a+b)^2$

third quadrant

* b) if $x < 0, y < 0$: $\dfrac{dy}{dx} = 3[(-x)(-y)]^{1/2} = 3(-x)^{1/2}(-y)^{1/2}$

$\int (-y)^{-1/2}\,dy = \int 3(-x)^{1/2}\,dx$

$-\dfrac{(-y)^{1/2}}{1/2} = 3(-)\dfrac{(-x)^{3/2}}{3/2} + C_1$

$(-y)^{1/2} = (-x)^{3/2} + \left(-\dfrac{C_1}{2}\right) \equiv C$

$-y = (C + (-x)^{3/2})^2$

$\boxed{y = -((-x)^{3/2} + C)^2}$

Combine: $y = \pm(|x|^{3/2} + C)^2$

Being careful. Too long for a quiz, but if we had to solve for the 1st initial condition, this would have been necessary.