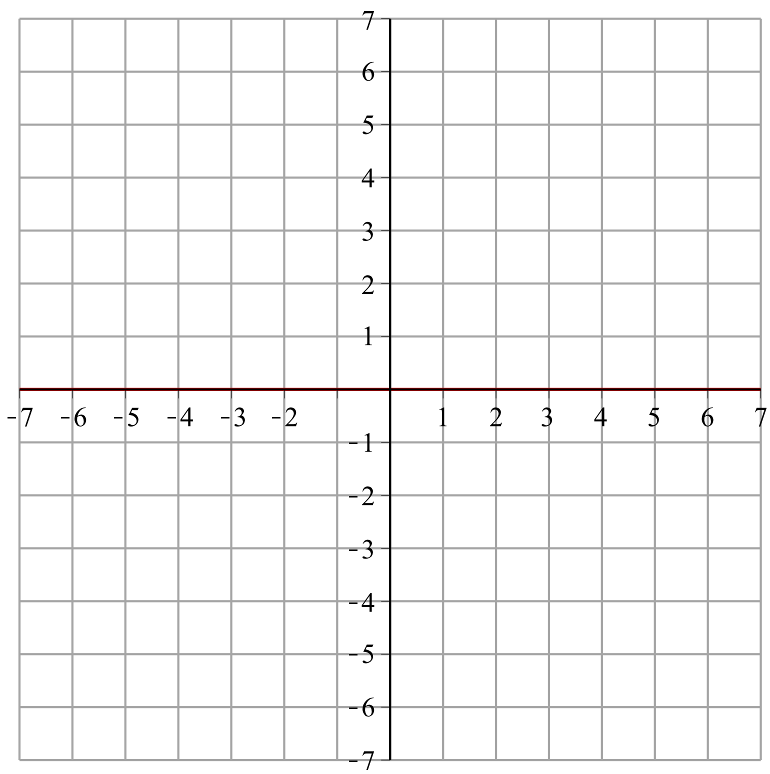
MAT2705-01/02 12F Test 2   Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions, determinants and matrix inverses.**

1. a) On the grid below, **draw in** arrows representing the vectors $\vec{v}_1 = \langle 1, 2\rangle$ and $\vec{v}_2 = \langle -2, 3\rangle$ and $\vec{v}_3 = \langle 4, 1\rangle$ and **label** them by their symbols. Then **draw in** the parallelogram that graphically expresses $\vec{v}_3$ as a linear combination of $\{\vec{v}_1, \vec{v}_2\}$. **Label** its two sides that intersect at the origin by the corresponding vectors they represent. **Extend** the basis vectors $\{\vec{v}_1, \vec{v}_2\}$ to the corresponding coordinate axes for (y_1, y_2) and **mark** the positive direction with an arrow head and the axis label.
From the grid, read off the coordinates (y_1, y_2) of $\vec{v}_3$ with respect to these two vectors (write them down) and **express** $\vec{v}_3$ as a linear combination of these vectors; **put this equation** at the tip of this vector. **Explain** how you got these numbers.
b) Now write down the matrix equation that enables you to express $\vec{v}_3$ as a linear combination of the other two vectors, solve that system using matrix methods, and then express $\vec{v}_3$ explicitly as a linear combination of those vectors.
c) Check your linear combination by expanding it out to get the original vector. Did you?
d) Does your matrix result agree with part a)?
e) Now using the new coordinate axes, **draw in** the vector $\vec{\boldsymbol{v}}_4$ whose new coordinates are $(y_1, y_2) = (-1, 1)$ and and **label** it by its symbol. Read off its old coordinates (x_1, x_2) from the grid. Do they agree with the linear combination $y_1\,\vec{v}_1 + y_2\,\vec{v}_2$?



2. For $A = \langle \vec{v}_1 | \vec{v}_2 | \vec{v}_3 | \vec{v}_4 \rangle$ with $\vec{v}_1 = \langle 2, 1, -3, 1 \rangle$, $\vec{v}_2 = \langle -2, 3, 1, 2 \rangle$, $\vec{v}_3 = \langle 6, -1, -7, 0 \rangle$, $\vec{v}_4 = \langle 2, 9, -7, 7 \rangle$.

a) Does $\vec{v}_5 = \langle -4, 2, 4, 0 \rangle$ lie in the span of this set? If so, show how it can be expressed in terms of them in the most general way. If not, show why not. Show all work to support your claim.

b) Does $\vec{v}_5 = \langle -4, 2, 4, 1 \rangle$ lie in the span of this set? If so, show how it can be expressed in terms of them in the most general way. If not, show why not. Show all work to support your claim.

c) Solve the equations $A\vec{x} = \vec{0}$ (state these matrix equations first), writing down the augmented matrix and its RREF form, identifying Leading and Free variables, and stating your result for $\vec{x}$.

d) From your general solution write down a basis $\vec{u}_1 = ..., ...$ of the solution space (in $\mathbb{R}^4$), using proper notation for a basis of vectors (as a set) in horizontal vector format $\langle a, ... \rangle$.

e) Write down the independent linear relationships among the original vectors that correspond to this basis.

f) Does the span of the original set of vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ represent a line, plane, a hyperplane or all of $\mathbb{R}^4$? Explain. State a basis for this subspace of $\mathbb{R}^4$ (which might be all of $\mathbb{R}^4$), using proper notation for a basis of vectors (as a set) in horizontal vector format $\langle a, ... \rangle$.

Sign and date the pledge at the end of your exam.

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: