

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. Mixing tank problem.

100 gal tank partially filled with saline solution: volume $V(t) = V_0 + (r_i - r_o) t$.

$x(t)$ is the amount of salt in the tank.

Saline solution is incoming, mixing, and flowing out.

$$\text{IVP: } x'(t) + \frac{r_o x(t)}{V_0 + (r_i - r_o) t} = r_i c_i, x(0) = 1$$

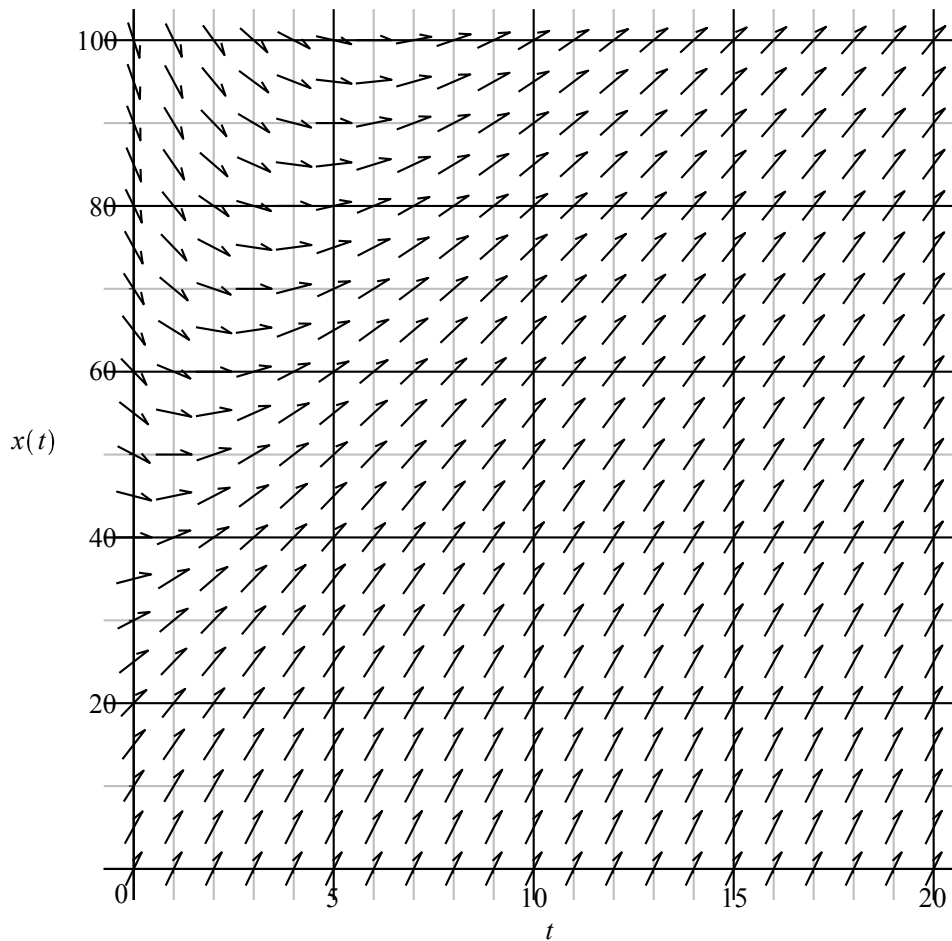
constants: $r_i = 10, r_o = 5, c_i = 1, V_0 = 20$

units: $V \sim \text{gal}, t \sim \text{min}, x \sim \text{lbs}$

- Write out the explicit differential equation using the $\frac{dx}{dt}$ notation inserting the values of the constants, so that it is ready to solve by hand.
- Insert the above Maple notation for the initial value problem into Maple with the values of the constants inserted and find the solution. Write it down for future reference.
- Hand draw in (carefully!) the solution of this differential equation satisfying the initial condition on the associated direction field on the reverse side of this sheet. Put a circled dot at the point corresponding to the initial condition.
- At what time T does the volume reach 100, so that the tank is full?
Estimate the value of x on this curve at the time T , putting a squared dot at this point.
- Use the linear DE solution algorithm to find the general solution of this differential equation by hand. Simplify it and box it.
- Find the solution of this differential equation which satisfies the given initial condition at $t = 0$. Simplify it and box it.
- Compare your numerical value of the exact solution for $x(T)$ with your graphical estimate. Do they reasonably agree?
- Does your initial value problem solution agree with Maple? Explain why or why not.

2. Now set $c_i = 0$ above, corresponding to pure water entering the tank.

- Find the general solution of this differential equation by hand using the separable technique. Simplify it and box it.
- Find the solution of this differential equation which satisfies the given initial condition. Simplify it and box it.
- How much salt is there in the tank when it is full?
- Does your final result for part b) agree with Maple's? Explain if they differ in any way.



Be sure to sign and date the pledge before handing in this test.

► solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:

Date: