

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. **BOX** final short answers. **LABEL** parts of problem. Keep answers **EXACT** (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions, determinants and matrix inverses.**

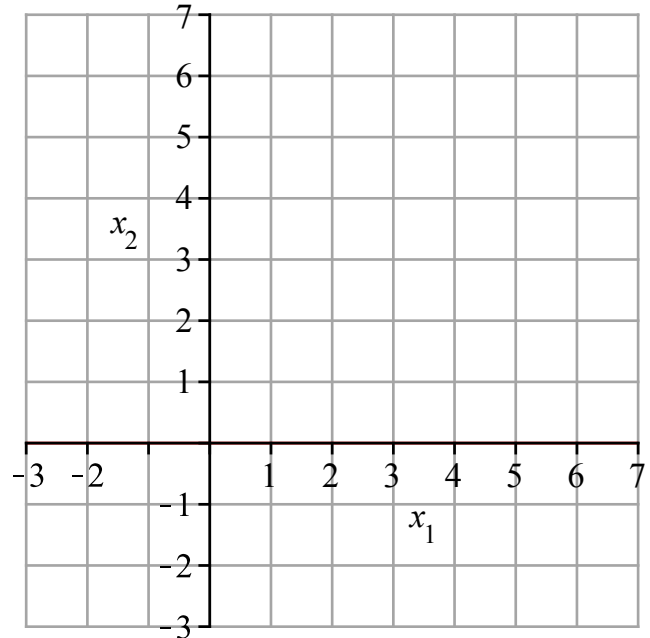
1. a) On the grid to the right, draw in arrows representing the vectors  $\vec{v}_1 = \langle -1, 1 \rangle$  and  $\vec{v}_2 = \langle 1, 3 \rangle$  and  $\vec{v}_3 = \langle 4, 4 \rangle$  and label them by their symbols. Then draw in the parallelogram that graphically expresses  $\vec{v}_3$  as a linear combination of  $\{\vec{v}_1, \vec{v}_2\}$ . Label its two sides that intersect at the origin by the corresponding vectors they represent. Extend the basis vectors  $\{\vec{v}_1, \vec{v}_2\}$  to the corresponding coordinate axes for  $(y_1, y_2)$  and mark the positive direction with an arrow head and the axis label.

From the grid, read off the coordinates  $(y_1, y_2)$  of  $\vec{v}_3$  with respect to these two vectors and express  $\vec{v}_3$  as a linear combination of them. Explain how you got these numbers.

b) Now write down the matrix equation that enables you to express  $\vec{v}_3$  as a linear combination of the other two vectors, solve that system using matrix methods, and then express  $\vec{v}_3$  explicitly as a linear combination of those vectors.

c) Check your linear combination by expanding it out to get the original vector. Did you?

d) Does your matrix result agree with part a)?



2. For  $A = \langle \vec{v}_1 | \vec{v}_2 | \vec{v}_3 \rangle$  with  $\vec{v}_1 = \langle 2, -1, 3 \rangle$ ;  $\vec{v}_2 = \langle 3, 4, -1 \rangle$ ;  $\vec{v}_3 = \langle 7, 2, 5 \rangle$ .

a) Does  $\vec{v}_4 = \langle 1, 5, -4 \rangle$  lie in the span of this set? If so, show how it can be expressed in terms of them. If not, show why not. Show all work to support your claim.

b) Does  $\vec{v}_4 = \langle 1, 5, 4 \rangle$  lie in the span of this set? If so, show how it can be expressed in terms of them. If not, show why not. Show all work to support your claim.

c) From your row reduction in either case, what conclusion can you draw about the linear independence or dependence of these three vectors? Explain.

3. a) Given the four vectors  $\vec{v}_1 = \langle 1, 5, 3, 2 \rangle$ ;  $\vec{v}_2 = \langle 2, -1, 4, 1 \rangle$ ;  $\vec{v}_3 = \langle -1, 6, -1, 1 \rangle$ ;  $\vec{v}_4 = \langle -5, 8, -9, -1 \rangle$  and their matrix as columns  $A = \langle \vec{v}_1 | \vec{v}_2 | \vec{v}_3 | \vec{v}_4 \rangle$  and  $\vec{x} = \langle x_1, x_2, x_3, x_4 \rangle$ , solve the equations  $A\vec{x} = \vec{0}$  (state these matrix equations first), writing down the augmented matrix and its RREF form, identifying Leading and Free variables, and stating your result for  $\vec{x}$ .

b) From your general solution write down a basis  $\vec{u}_1 = \dots, \dots$  of the solution space (in  $\mathbb{R}^4$ ).

c) Write down the independent linear relationships among the original vectors that correspond to this basis.

d) Does the span of the original set of vectors  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$  represent a line, plane, a hyperplane or all of  $\mathbb{R}^4$ ? Explain. State a basis for this subspace of  $\mathbb{R}^4$  (which might be all of  $\mathbb{R}^4$ ).

**Sign and date pledge on reverse side at the end of your exam.**

## ► solution

## ▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:

Date: