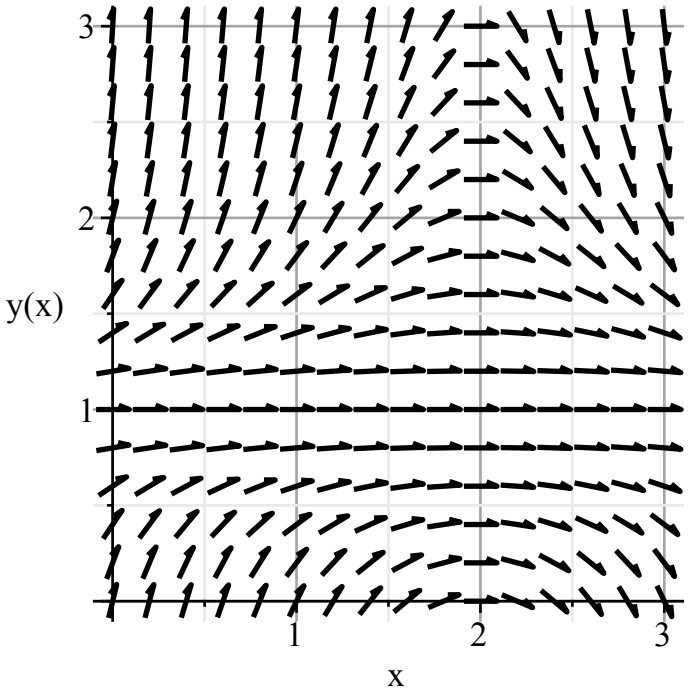
MAT2705-03/04 11S Test 1 Print Name (Last, First) _____|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $2(y-1)^2 + (x-2)^{-1}\,\frac{dy}{dx} = 0,\quad y(1) = 2$

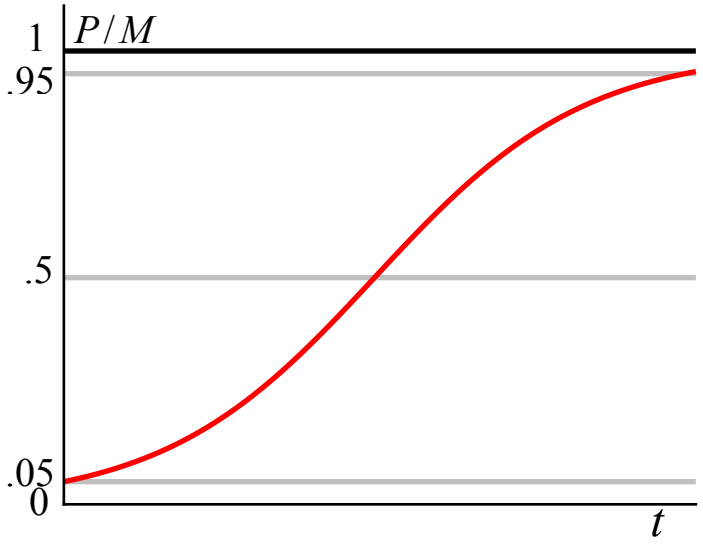
a) Hand draw in the solution of this differential equation satisfying the initial condition on the associated direction field to the right. Put a circled dot at the point corresponding to the initial condition. Put in a squared dot at the point on the curve for which $x = 0$. Estimate the value of $y(0)$.
b) Find the general solution of this differential equation by hand. Simplify it and box it.
c) Find the solution of this differential equation which satisfies the given initial condition. Simplify it and box it.
d) Evaluate your IVP solution at $x = 0$. Is your result consistent with part a)? Explain.
e) Does your initial value problem solution agree with Maple? Explain why or why not.
f) What is the solution which instead satisfies the initial condition $y(1) = 1$?



2. $\frac{1}{t}\,\frac{dx}{dt} = e^{\frac{t^2}{2}} - x,\ x(0) = 1$.

a) Find the general solution of this differential equation by hand. Simplify it and box it.
b) Find the solution of this differential equation which satisfies the given initial condition. Simplify it and box it.
c) The correct solution of the IVP satisfies $x^2 - (x'/t)^2 = 1$. Does your solution?

3. A logistic population curve is characterized by the exponential decay $\frac{M-P}{P} = C\,e^{-0.04t}$ of the ratio of the difference between the maximum population M and the population P and the population itself.
a) What is the value of the characteristic time τ for this decay process?
b) If the initial ratio is 19 to 1 (namely $P = 0.05\,M$), how long does it take to reach the value of 1 to 19 (namely $P = 0.95\,M$)?
c) How many multiples of the characteristic time does this represent? (2 decimal places are sufficient.)



d) **Optional. Not extra credit.**
How many multiples would it take to go instead from the 1 percent line to the 99 percent line?
[change $0.05 \rightarrow 0.01$ above]

Be sure to sign and date the pledge on page 2 before handing in this test.

solution

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: