MAT2705-03/04 11F Quiz 8 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $x_1'(t) = -2\,x_1(t)$, $x_2'(t) = 7\,x_1(t) - 9\,x_2(t) - 7\,x_3(t)$, $x_3'(t) = -2\,x_3(t)$,
$x_1(0) = 2$, $x_2(0) = -2$, $x_3(0) = 1$.

a) Rewrite this system of DEs **and** its initial conditions explicitly in matrix form for the vector variable $\vec{\boldsymbol{x}} = \langle x_1, x_2, x_3 \rangle$ as a column matrix (using the actual matrices, not their symbols), identifying the coefficient matrix A.

b) For this A, using Maple write down the eigenvalues λ_1, λ_2, λ_3 and corresponding matrix of eigenvectors $B = \langle \vec{\boldsymbol{b}}_1 | \vec{\boldsymbol{b}}_2 | \vec{\boldsymbol{b}}_3 \rangle$ that it provides you.

c) Pick one eigenvector $\vec{\boldsymbol{b}}_i$ and evaluate by hand the product $A\,\vec{\boldsymbol{b}}_i$ and compare with $\lambda_i\,\vec{\boldsymbol{b}}_i$. Do they agree?

d) Use technology to evaluate and write down the inverse matrix B^{-1} and use Maple to evaluate the matrix product $A_B = B^{-1} A\,B$. Write down this result. Does it evaluate correctly to the diagonalized matrix with the eigenvalues in the correct order?

e) Given that $\vec{x} = B\,\vec{y}$, if $\vec{x}(0) = \langle 2, -2, 1 \rangle$, find $\vec{y}(0)$. Show how you did this.

f) Express the new components of the system of differential equations: $\vec{y}\,' = A_B\,\vec{y}$, solve them, then backsubsitute into $\vec{x} = B\,\vec{y}$.

g) Impose the initial conditions and express your solution for $\vec{x}$ both in matrix form, multiplying out to obtain the scalar solutions for the individual unknowns, as well as in the linear combination vector form which shows the three modes, and then combine the two modes associated with the repeated eigenvalue to get a linear combination of only 2 independent vectors.

h) The resulting motion takes place in a plane. State a basis for that plane.

i) What are the two characteristic times for the exponential decays? Plot your solutions for $t = 0..T$ where T is 5 times the larger characteristic time. Make a rough sketch of what you see, labeling the three curves by their variables, and labeling the axes with tickmarks and the norizontal one with its variable name. Two of the curves merge in your plot. Their difference decays with the smaller characteristic time constant. Does the interval over which they merge roughly agree with 5 times that smaller time constant?

j) Only one of these curves has a local extremum. Find its coordinates exactly and approximately to 3 significant digits. Is your result consistent with your plot?

▶ solution