MAT2705-03/04 11F Quiz 6 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $y''-3y'+2y=0, y(0)=7, y'(0)=6$.

a) Verify that the general solution $y(x)=c_1 e^x + c_2 e^{2x}$ actually solves the DE.

b) Using 2x2 matrix methods find the solution which satisfies the initial conditions, showing all work.

c) Use technology to plot your result for $x = 0 .. 2.2$ and make a rough sketch of what you see, labeling the axes with variable names and key tickmarks on your sketch.

d) Use calculus to determine ***exactly*** by hand (no decimal pints, use rules of exponents and logs! simplify your result for y to a simple expression) the x and y values of the obvious maximum point on the graph and then their approximate values to 4 decimal places. Do the numbers you found agree with what your eyes see in the technology plot? [Yes or no, with an explanation would be a good response.]

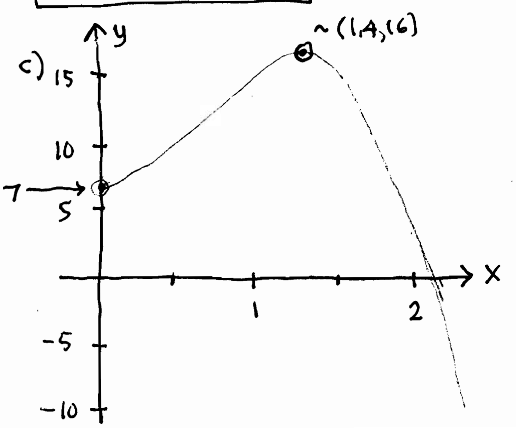
e) Use Maple to make sure your solution of the initial value problem is correct. Does Maple confirm your result?

▶ solution

① a)
$$2\,[y = c_1 e^x + c_2 e^{2x}]$$
$$-3\,[y' = c_1 e^x + 2c_2 e^{2x}]$$
$$1\,[y'' = c_1 e^x + 4c_2 e^{2x}]$$
$$y''-3y'+2y = \underbrace{(2-3+1)}_{0} c_1 e^x + \underbrace{(2-6+4)}_{0} e^{2x} = 0 \checkmark$$

b)
$$y(0) = c_1 + c_2 = 7$$
$$y'(0) = c_1 + 2c_2 = 6$$
$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$
row reduce or use inverse:
$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 6 \end{bmatrix} = \frac{1}{2-1} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$
$$= \begin{bmatrix} 14-6 \\ -7+6 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$
$$\boxed{y = 8e^x - e^{2x}}$$

c)



d)
$$y' = \frac{8e^x - 2e^{2x} = 0}{2e^x}$$
$$4 - e^x = 0 \to e^x = 4$$
$$x = \ln 4$$
$$y = 8e^{\ln 4} - e^{2\ln 4}$$
$$= 8(4) - 4^2 = 32 - 16 = 16 \text{ exactly!}$$
max at $\boxed{(x,y) = (\ln 4, 16) \approx (1.3863, 16.0000)}$

very consistent with graph!

e) Maple agrees!