MAT2705-03/04 11F Quiz 5 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **[You should use technology for row reductions and determinants. Report results you find in this way: value of det, reduced matrix.]**

1. $\vec{v}_1 = \langle 3, 1, -1\rangle$, $\vec{v}_2 = \langle -3, -2, 1\rangle$, $\vec{v}_3 = \langle -3, -5, 1\rangle$, $\vec{v}_4 = \langle 6, 2, -2\rangle$, $\vec{v}_5 = \langle -6, -7, 2\rangle$

a) Express v_5 as a linear combination of the remaining 4 vectors, in the most general way. [Final answer: $\vec{v}_5 = \ldots\, \vec{v}_1 + \ldots$]

b) Check that this general linear combination that you find actually evaluates to $\vec{v}_5$.

c) Now express the coefficient vector you found as a linear combination of constant vectors multiplied by arbitrary parameters plus a single constant additive vector.

d) From part c), identify the independent linear relationships among these 4 vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ (i.e., what ***independent*** linear combinations of these vectors equal the zero vector?). Write out these relationships individually.

2. $5x_1 + 3x_2 = 2$, $-3x_1 + -2x_2 = -1$ a) Write this linear system in the matrix form $\mathbf{A}\vec{x} = \vec{b}$.

b) Write down the inverse coefficient matrix using technology or your memory if good enough, but then verify that its product with **A** is the identity matrix. Show the matrix multiplication steps by hand (sums of products before simplifying) to prove that you can actually multiply simple matrices.

c) Now solve this matrix equation for the column matrix $\vec{x}$ using the inverse matrix, and then write out the individual scalar solutions of the original system for each individual variable.

d) Check by backsubstitution into the original two equations that your solution is actually a solution.

► solution

1 a) $x_1 \begin{bmatrix}3\\1\\-1\end{bmatrix} + x_2\begin{bmatrix}-3\\-2\\1\end{bmatrix} + x_3\begin{bmatrix}-3\\-5\\1\end{bmatrix} + x_4\begin{bmatrix}+6\\2\\-2\end{bmatrix}$

$= \begin{bmatrix}3&-3&-3&6\\1&-2&-5&2\\-1&1&1&-2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}-6\\-7\\2\end{bmatrix} \rightarrow$

$$\begin{bmatrix}3&-3&-3&6&-6\\1&-2&-5&2&-7\\-1&1&1&-2&2\end{bmatrix} \xrightarrow{\text{rref}} \begin{array}{c} \begin{array}{cccc} L & L & F & F \\ x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix}1&0&3&2&3\\0&1&4&0&5\\0&0&0&0&0\end{bmatrix}\end{array}$$

$x_1 + 3x_3 + 2x_4 = 3 \rightarrow x_1 = -3t_1 - 2t_2 + 3$

$x_2 + 4x_3 = 5 \rightarrow x_2 = -4t_1 + 5$

$0 = 0$ $\quad x_3 = t_1$

$x_3 = t_1,\ x_4 = t_2$ $\quad x_4 = t_2$

$$\therefore \boxed{\begin{bmatrix}-6\\-7\\2\end{bmatrix} = (3-3t_1-2t_2)\begin{bmatrix}3\\1\\-1\end{bmatrix} + (5-4t_1)\begin{bmatrix}-3\\-2\\1\end{bmatrix} + t_1\begin{bmatrix}-3\\-5\\1\end{bmatrix} + t_2\begin{bmatrix}6\\2\\-2\end{bmatrix}}$$

b) $= \begin{bmatrix}3(3-3t_1-2t_2) - 3(5-4t_1) - 3t_1 + 6t_2\\ (3-3t_1-2t_2) - 2(5-4t_1) - 5t_1 + 2t_2 \\ -(3-3t_1-2t_2) + (5-4t_1) + t_1 - 2t_2\end{bmatrix}$

$= \begin{bmatrix}9-15 + t_1(-9+12-3) + t_2(-6+6)\\ 3-10 + t_1(-3+8-5) + t_2(-2+2)\\ -3+5 + t_1(3-4+1) + t_2(2-2)\end{bmatrix} = \begin{bmatrix}-6\\-7\\2\end{bmatrix}$ ✓

c) $\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}3-3t_1-2t_2\\5-4t_1\\t_1\\t_2\end{bmatrix} = \begin{bmatrix}3\\5\\0\\0\end{bmatrix} + \underbrace{t_1\begin{bmatrix}-3\\-4\\1\\0\end{bmatrix} + t_2\begin{bmatrix}-2\\0\\0\\1\end{bmatrix}}_{\text{hom soln, coeffs} \rightarrow \text{lin relationships}}$

d) $\boxed{-3\vec{v}_1 - 4\vec{v}_2 + \vec{v}_3 = \vec{0},\quad -2\vec{v}_1 + \vec{v}_4 = \vec{0}}$

2. $\underbrace{\begin{bmatrix}5&3\\-3&-2\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x_1\\x_2\end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix}2\\-1\end{bmatrix}}_{\vec{b}}$ $\qquad A^{-1} = \frac{1}{-10+9}\begin{bmatrix}-2&-3\\3&5\end{bmatrix} = \begin{bmatrix}2&3\\-3&-5\end{bmatrix}$

$A^{-1}A = \begin{bmatrix}2&3\\-3&-5\end{bmatrix}\begin{bmatrix}5&3\\-3&-2\end{bmatrix} = \begin{bmatrix}2(5)+3(-3) & 2(3)+3(-2)\\ -3(5)+5(3) & -3(3)+5(2)\end{bmatrix}$

$= \begin{bmatrix}10-9 & 6-6\\ 15-15 & -9+10\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix}$ ✓

$\vec{x} = \begin{bmatrix}x_1\\x_2\end{bmatrix} = A^{-1}\vec{b} = \begin{bmatrix}2&3\\-3&-5\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}2(2)-3(1)\\-3(2)+5(1)\end{bmatrix} = \begin{bmatrix}4-3\\5-6\end{bmatrix}$

$= \begin{bmatrix}1\\-1\end{bmatrix}$ soln.

d) $5(1) + 3(-1) \stackrel{?}{=} 2$, $\quad -3(1) + -2(-1) \stackrel{?}{=} -1$

$= 5-3$ $\qquad = -3+2$

$= 2$ ✓ $\qquad = -1$ ✓