MAT2705-03/04 11F Quiz 4 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Word Problem. The time rate of change of an alligator population P in a swamp is proportional to the square of P. The swamp contained a dozen alligators in 1995, two dozen in 2005. When will there be four dozen alligators in the swamp? What happens thereafter?

Proceed to respond as follows:

a) Write down the differential equation and the initial condition in proper notation as an initial value problem, together with the additional condition needed to determine the constant of proportionality.

b) Find the general solution of the differential equation by hand.

c) Find the solution satisfying the initial condition.

d) Use the additional condition to finish specifying the population function as a unique function of the time.

e) Answer the first question posed in a complete sentence, showing all supporting work.

f) Make a diagram showing your solution and annotate it with the information about P given in the problem statement. Looking at your graph, what happens to the population as it continues past four dozen? Is this a realistic model? Explain.

To check with Maple, you need to use function notation: $P(t)$ and $P'(t)$ in the differential equation.

▶ solution

a) $\frac{dP}{dt} \propto P^2 \rightarrow \boxed{\frac{dP}{dt} = kP^2, \quad P(0) = 12, \quad P(10) = 24}$ $\quad t=0 \leftrightarrow 1995$

units: years

b) Separate, Integrate: $\int P^{-2}\,dP = \int k\,dt$

$$\frac{P^{-1}}{-1} = kt + C_1$$

$$P^{-1} = -kt - C_1$$

$$\boxed{P = \frac{1}{-kt - C_1} = \frac{1}{C - kt}}$$ $C = -C_1$ (not necessary)

c) $12 = P(0) = \frac{1}{C} \rightarrow C = 1/12$

$$\boxed{P = \frac{1}{\frac{1}{12} - kt} = \frac{12}{1 - 12kt}}$$ "simpler"

d) $24 = P(10) = \frac{12}{1 - 120k}$

$1 - 120k = \frac{12}{24} = \frac{1}{2}$

$\frac{1}{2} = 120k \quad k = \frac{1}{240}$

$$\boxed{P = \frac{12}{1 - \frac{12t}{240}} = \frac{12}{1 - \frac{t}{20}} = \boxed{\frac{240}{20 - t}}}$$ simplest.

e) $48 = \frac{240}{20 - t}$

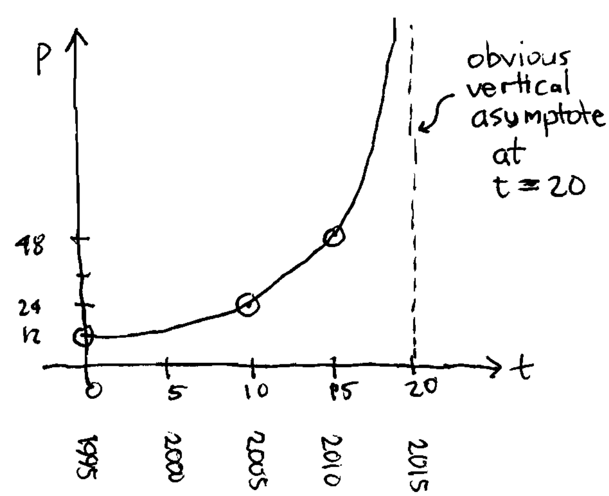
$20 - t = \frac{240}{48} = 5$

$t = 20 - 5 = 15$

$1995 + 15 = 2010$

There will be four dozen alligators in 2010.

f)



The population cannot go to ∞ at $t = 20$ so the model must break down before then. The model cannot be realistic near this time.