

① a)  $9x'' + 6x' + 37x = F(t) / 9 \rightarrow$

$$x'' + \frac{2}{3}x' + \frac{37}{9}x = \frac{1}{9}F(t)$$

$$\underbrace{\frac{2}{3}}_{R_0} \quad \underbrace{\frac{37}{9}}_{\omega_0^2} \quad \omega_0 = \sqrt{\frac{37}{9}} = \frac{\sqrt{37}}{3} \approx 2.028$$

$$R_0 = 2/3 \approx 0.667 \quad Q = \omega_0 R_0 = \frac{\sqrt{37}}{2} \approx 3.041$$

$$T_0 = 1/\omega_0 = 3/2 = 1.5 \quad T_0 = \frac{2\pi}{\omega_0} = \frac{6\pi}{\sqrt{37}} \approx 3.099$$

b)  $x \sim e^{rt}: 9r^2 e^{rt} + 6r e^{rt} + 37e^{rt} = 0$

$$9r^2 + 6r + 37 = 0, \quad r = \frac{-6 \pm \sqrt{36 - 4 \cdot 9 \cdot 37}}{2 \cdot 9}$$

$$= \frac{-6 \pm 36i}{2 \cdot 9} = -\frac{1}{3} \pm 2i$$

$$e^{rt} = e^{-\frac{t}{3}} e^{\pm 2i} = e^{-\frac{t}{3}} (\cos 2t \pm i \sin 2t) \quad \text{complex basis} \rightarrow$$

$$\rightarrow e^{-\frac{t}{3}} \cos 2t, e^{-\frac{t}{3}} \sin 2t \quad \text{real basis of soln space}$$

Gen soln:  $x = e^{-t/3} (c_1 \cos 2t + c_2 \sin 2t)$

$$x' = -\frac{1}{3} e^{-t/3} (c_1 \cos 2t + c_2 \sin 2t) + e^{-t/3} (-2c_1 \sin 2t + 2c_2 \cos 2t)$$

$$R_1 = \frac{1}{3} \approx 0.333, \quad \omega_1 = 2$$

$$T_1 = 1/R_1 = 3 \quad T_1 = 2\pi/\omega_1 = \pi \approx 3.142$$

c)  $9x'' + 6x' + 37x = 5 \cos 2t$

$$37 [X_p = c_3 \cos 2t + c_4 \sin 2t]$$

$$6 [X_p' = -2c_3 \sin 2t + 2c_4 \cos 2t]$$

$$9 [X_p'' = -4c_3 \cos 2t - 4c_4 \sin 2t]$$

$$9X_p'' + 6X_p' + 37X_p = (37-36)c_3 + 12c_4 \cos 2t = 5 \cos 2t$$

$$+ [-12c_3 + (37-36)c_4] \sin 2t$$

$$c_3 + 12c_4 = 5$$

$$-12c_3 + c_4 = 0 \quad \begin{bmatrix} 1 & 12 \\ -12 & 1 \end{bmatrix} \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \frac{1}{145} \begin{bmatrix} 1 & -12 \\ 12 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \frac{5}{529} \begin{bmatrix} 1 \\ 12 \end{bmatrix} = \begin{bmatrix} 1/29 \\ 12/29 \end{bmatrix}$$

$$X = X_h + X_p = e^{-t/3} (c_1 \cos 2t + c_2 \sin 2t) + \frac{1}{29} [\cos 2t + 12 \sin 2t]$$

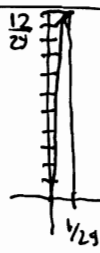
$$x' = -\frac{1}{3} e^{-t/3} (c_1 \cos 2t + c_2 \sin 2t) + \frac{1}{29} [-2 \sin 2t + 24 \cos 2t]$$

$$+ e^{-t/3} (-2c_1 \sin 2t + 2c_2 \cos 2t)$$

$$x(0) = c_1 + 1/29 = 0 \rightarrow c_1 = -1/29$$

$$x'(0) = -\frac{c_1}{3} + 2c_2 + 24 = 0 \rightarrow c_2 = \frac{1}{3} \left( \frac{1}{29} \right) - \frac{24}{29} = \frac{-73}{174}$$

$$x = e^{-t/3} \left( \frac{-73}{174} \sin 2t - \frac{1}{29} \cos 2t \right) + \frac{1}{29} [\cos 2t + 12 \sin 2t]$$



$$A = \frac{1}{29} \sqrt{1+12^2} = \frac{\sqrt{145}}{29} = \frac{\sqrt{5 \cdot 29}}{29} = \frac{\sqrt{5}}{\sqrt{29}} \approx 0.415$$

$$\delta = \arctan 12 \approx 1.498 \approx 85.2^\circ \approx 0.237 \text{ cycles}$$

$$\text{so } X_{ss} = \frac{\sqrt{5}}{29} \cos(2t - \arctan 12)$$

d)  $9x'' + 6x' + 37x = F_0 \cos \omega t$

$$37 [X_p = c_3 \cos \omega t + c_4 \sin \omega t]$$

$$6 [X_p' = -\omega c_3 \sin \omega t + \omega c_4 \cos \omega t]$$

$$9 [X_p'' = -\omega^2 c_3 \cos \omega t - \omega^2 c_4 \sin \omega t]$$

$$9X_p'' + 6X_p' + 37X_p = [(37-9\omega^2)c_3 + 6\omega c_4] \cos \omega t$$

$$+ [-6\omega c_3 + (37-9\omega^2)c_4] \sin \omega t = F_0 \cos \omega t$$

$$\begin{bmatrix} 37-9\omega^2 & 6\omega \\ -6\omega & 37-9\omega^2 \end{bmatrix} \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \frac{1}{(37-9\omega^2)^2 + 36\omega^2} \begin{bmatrix} 37-9\omega^2 & -6\omega \\ 6\omega & 37-9\omega^2 \end{bmatrix} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$= \frac{F_0}{(37-9\omega^2)^2 + 36\omega^2} \begin{bmatrix} 37-9\omega^2 \\ 6\omega \end{bmatrix}$$

$$X_{ss} = \frac{F_0}{(37-9\omega^2)^2 + 36\omega^2} \left[ (37-9\omega^2) \cos \omega t + 6\omega \sin \omega t \right]$$

e)  $A(\omega) = \frac{F_0 \sqrt{(37-9\omega^2)^2 + 36\omega^2}}{(37-9\omega^2)^2 + 36\omega^2} = \frac{F_0}{\sqrt{(37-9\omega^2)^2 + 36\omega^2}}$

$$= \frac{F_0 (1369 - 630\omega^2 + 81\omega^4)^{-1/2}}{\sqrt{(37-9\omega^2)^2 + 36\omega^2}}$$

$$A(2) = \frac{5}{\sqrt{(37-36)^2 + 36 \cdot 4}} = \frac{5}{\sqrt{144+144}} = \frac{5}{\sqrt{288}} = \frac{5}{12\sqrt{2}}$$

$$= \frac{\sqrt{145}}{29} \quad A(0) = F_0/37$$

$$0 = A'(\omega) = F_0 \left(-\frac{1}{2}\right) (\dots)^{-3/2} (2(37-9\omega^2)(-18\omega) + 72\omega)$$

$$0 = 36\omega (-37+9\omega^2 + 2) = 36\omega (9\omega^2 - 35)$$

$$\omega = 0, \quad \omega_p = \sqrt{35/9} = \sqrt{35}/3 \approx 1.972$$

$$A(\omega_p) = F_0 (37-35)^2 + 36 \cdot (35/9))^{-1/2}$$

$$= F_0 (4 + 4(35))^{-1/2} = F_0 (4 \cdot 36)^{-1/2} = F_0/12$$

$$A(\omega_p)/F_0 = 1/12 \approx 0.0833$$

$$F_0 = 5: A(2) = \frac{5}{\sqrt{(37-9 \cdot 4)^2 + 36 \cdot 4}}^{-1/2}$$

$$= 5(145)^{-1/2} = \frac{5}{\sqrt{5 \cdot 29}} = \frac{1}{\sqrt{29}} \checkmark$$

$$\frac{A(\omega_p)}{A(0)} = \frac{F_0/12}{F_0/37} = \frac{37}{12} \approx 3.083 \leftarrow \text{pretty close!}$$

$$Q \approx 3.041 \leftarrow$$

② a)  $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}' = \underbrace{\begin{bmatrix} -1/2 & 0 & 1/2 \\ 1/2 & -1/5 & 0 \\ 0 & 1/5 & -1/2 \end{bmatrix}}_A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix}$

b)  $0 = |A - \lambda I| = \begin{vmatrix} -1/2 - \lambda & 0 & 1/2 \\ 1/2 & -1/5 - \lambda & 0 \\ 0 & 1/5 & -1/2 - \lambda \end{vmatrix} = -\lambda^3 - \frac{6}{5}\lambda^2 - \frac{9}{20}\lambda$   
 $= -\lambda(\lambda^2 + \frac{6}{5}\lambda + \frac{9}{20})$

$\lambda = 0, -\frac{3}{5} \pm \frac{3}{10}i$

$\lambda_1 = \lambda = 0$ :  $A - \lambda I = \begin{bmatrix} -1/2 & 0 & 1/2 \\ 1/2 & -1/5 & 0 \\ 0 & 1/5 & -1/2 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -5/2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_3 = t, x_1 = +t, x_2 = +5/2 t$

$\langle x_1, x_2, x_3 \rangle = \langle t, 5/2 t, t \rangle = t \langle 1, 5/2, 1 \rangle = \vec{b}_1$

$\lambda_2 = \lambda = -\frac{3}{5} + \frac{3}{10}i$ :  $A - \lambda I = \begin{bmatrix} -1/2 + \frac{3}{5} - \frac{3}{10}i & 0 & 1/2 \\ 1/2 & -1/5 + \frac{3}{5} - \frac{3}{10}i & 0 \\ 0 & 1/5 & -1/2 + \frac{3}{5} - \frac{3}{10}i \end{bmatrix}$

$\xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & \frac{1}{2}(1+3i) \\ 0 & 1 & \frac{1}{2}(1-3i) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_3 = t, x_1 = -\frac{1}{2}(1+3i)t, x_2 = -\frac{1}{2}(1-3i)t$

$\langle x_1, x_2, x_3 \rangle = \langle t, -\frac{1}{2}(1+3i)t, -\frac{1}{2}(1-3i)t \rangle = t \langle 1, -\frac{1}{2}(1+3i), -\frac{1}{2}(1-3i) \rangle = \vec{b}_2 = \vec{b}_3, \lambda_3 = \bar{\lambda}_2$

$B = \langle \vec{b}_1, \vec{b}_2, \vec{b}_3 \rangle = \begin{bmatrix} 1 & -1/2(1+3i) & -1/2(1-3i) \\ 5/2 & -1/2(1-3i) & -1/2(1+3i) \\ 1 & 1 & 1 \end{bmatrix}$

$(\vec{x} = B\vec{y}, \vec{y} = B^{-1}\vec{x}) \rightarrow \vec{x}' = A\vec{x} \rightarrow \vec{y}' = AB\vec{y}$

$AB = B^{-1}AB = \begin{bmatrix} -1/2 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & -1/2 \end{bmatrix}$

$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 y_1 \\ (-\frac{3}{5} + \frac{3}{10}i) y_2 \\ (-\frac{3}{5} - \frac{3}{10}i) y_3 \end{bmatrix}$

$y_1 = c_1 e^{-3t/5} e^{3it/10}$

$y_2 = c_2 e^{-3t/5} e^{-3it/10}$

$y_3 = c_3 e^{-3t/5} e^{-3it/10}$

$\vec{x} = c_1 \vec{b}_1 + c_2 e^{-3t/5} e^{3it/10} \vec{b}_2 + c.c.$

find Re, Im parts for real basis

② b)  $e^{-\frac{3t}{5}} (\cos \frac{3t}{10} + i \sin \frac{3t}{10}) \begin{bmatrix} -\frac{1}{2}(1+3i) \\ -\frac{1}{2}(1-3i) \\ 1 \end{bmatrix}$   
 $= e^{-\frac{3t}{5}} \begin{bmatrix} -\frac{1}{2}(\cos \frac{3t}{10} - 3\sin \frac{3t}{10} + 3i(\cos \frac{3t}{10} + i \sin \frac{3t}{10})) \\ -\frac{1}{2}(\cos \frac{3t}{10} + 3\sin \frac{3t}{10} - 3i(\cos \frac{3t}{10} + i \sin \frac{3t}{10})) \\ \cos \frac{3t}{10} + i \sin \frac{3t}{10} \end{bmatrix}$   
 $= e^{-\frac{3t}{5}} \underbrace{\begin{bmatrix} -\frac{1}{2}(\cos \frac{3t}{10} - 3\sin \frac{3t}{10}) \\ -\frac{1}{2}(\cos \frac{3t}{10} + 3\sin \frac{3t}{10}) \\ \cos \frac{3t}{10} \end{bmatrix}}_{\vec{x}_1} + i e^{-\frac{3t}{5}} \underbrace{\begin{bmatrix} -\frac{1}{2}(3\cos \frac{3t}{10} + \sin \frac{3t}{10}) \\ -\frac{1}{2}(-3\cos \frac{3t}{10} + \sin \frac{3t}{10}) \\ \sin \frac{3t}{10} \end{bmatrix}}_{\vec{x}_2}$

$\{\vec{x}_1, \vec{x}_2\}$  real basis of this subspace of soln space so:

$\vec{x} = c_1 \begin{bmatrix} 1 \\ 5/2 \\ 1 \end{bmatrix} + c_2 \vec{x}_1 + c_3 \vec{x}_2$

$\begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix} = \vec{x}(0) = c_1 \begin{bmatrix} 1 \\ 5/2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 5/2 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} -1/2 \\ -1/2 \\ 3/2 \end{bmatrix}$   
 $= \begin{bmatrix} 1 & -1/2 & -3/2 \\ 5/2 & -1/2 & 3/2 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} \downarrow \\ \downarrow \\ \downarrow \end{bmatrix}^{-1} \begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ -8 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 5/2 \\ 1 \end{bmatrix} - 4e^{-\frac{3t}{5}} \begin{bmatrix} -1/2(-3s) \\ -1/2(c+3s) \\ c \end{bmatrix} - 8e^{-\frac{3t}{5}} \begin{bmatrix} -1/2(3c+s) \\ -1/2(-3c+s) \\ s \end{bmatrix}$

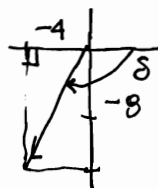
$= \begin{bmatrix} 4 + e^{-\frac{3t}{5}}(2c - 6s + 12c + 4s) \\ 10 + e^{-\frac{3t}{5}}(2c + 6s - 12c + 4s) \\ 4 - 4e^{-\frac{3t}{5}}(c + 2s) \end{bmatrix}$

$= \begin{bmatrix} 4 + e^{-\frac{3t}{5}}(14\cos \frac{3t}{10} - 2\sin \frac{3t}{10}) \\ 10 + e^{-\frac{3t}{5}}(-10\cos \frac{3t}{10} + 10\sin \frac{3t}{10}) \\ 4 - 4e^{-\frac{3t}{5}}(\cos \frac{3t}{10} + 2\sin \frac{3t}{10}) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

agrees with Maple!

c)  $\vec{x}_{\infty} = \begin{bmatrix} 4 \\ 10 \\ 4 \end{bmatrix}$  since  $e^{-\frac{3t}{5}} \rightarrow 0$  as  $t \rightarrow \infty$

d)  $x_3 - x_{3\infty} = -4e^{-\frac{3t}{5}} (\cos \frac{3t}{10} + 2\sin \frac{3t}{10})$



$A = 4\sqrt{1+2^2} = 4\sqrt{5} \approx 8.944$

$\delta = -\pi + \arctan 2 \approx -0.324$  cycles  
 $\approx -2.034 \approx -116.6^\circ$

$\tau = \frac{5}{3}, 5\tau = \frac{25}{3} \approx 8\frac{1}{3} \approx 10$  for viewing window

$$\textcircled{3} \text{ a) } \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & 4 \\ 6 & -5 \end{bmatrix}}_A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$0 = |A - \lambda I| = \begin{vmatrix} -3-\lambda & 4 \\ 6 & -5-\lambda \end{vmatrix} = (\lambda+3)(\lambda+5) - 24 \\ = \lambda^2 + 8\lambda + 15 - 24 \\ = \lambda^2 + 8\lambda - 9$$

$$\lambda = 1, -9 \quad \lambda_1 = 1, \lambda_2 = -9$$

$$\lambda = 1: A - I = \begin{bmatrix} -3-1 & 4 \\ 6 & -5-1 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ 6 & -6 \end{bmatrix} \xrightarrow{L \ F} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_2 = t, x_1 = t \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} t \\ t \end{bmatrix}}_{b_1} = t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda = -9: A + 9I = \begin{bmatrix} -3+9 & 4 \\ 6 & -5+9 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 6 & 4 \end{bmatrix} \xrightarrow{L \ F} \begin{bmatrix} 1 & 2/3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_2 = t, x_1 = -\frac{2}{3}t \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2/3 t \\ t \end{bmatrix} = t \begin{bmatrix} -2/3 \\ 1 \end{bmatrix} = 3t \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} \quad B^{-1} = \frac{1}{5} \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix} \quad A_B = B^{-1}AB = \begin{bmatrix} 1 & 0 \\ 0 & -9 \end{bmatrix}$$

$$\text{c) } \vec{x}' = A\vec{x} \rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} B\vec{y} \\ B^{-1}\vec{x} \end{pmatrix} \rightarrow \vec{y}' = A_B\vec{y}$$

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ -9y_2 \end{bmatrix} \quad \begin{matrix} y_1' = y_1 & y_1 = c_1 e^t \\ y_2' = -9y_2 & y_2 = c_2 e^{-9t} \end{matrix}$$

$$\begin{bmatrix} -1 \\ 4 \end{bmatrix} = \vec{x}(0) = B \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \rightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3+8 \\ -1+4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} e^t \\ e^{-9t} \end{bmatrix} = e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{-9t} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} e^t - 2e^{-9t} \\ e^t + 3e^{-9t} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{b) } \begin{bmatrix} 0 \\ 5 \end{bmatrix} = B \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \rightarrow \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

↑ oops, bob forgot to edit this to be  $\langle -1, 4 \rangle$  as in IVP but no matter. makes sense anyway.

$$\text{d) } x_2 = e^t + 3e^{-9t}$$

$$x_2' = e^t - 27e^{-9t} = 0 \rightarrow e^{10t} - 27 = 0$$

$$10t = \ln 27 \quad t = \frac{1}{10} \ln 27 = \frac{1}{10} \ln 3^3 = \frac{3 \ln 3}{10} \approx 0.328$$

$$x_2 = e^{\frac{1}{10} \ln 27} + 3e^{-\frac{9}{10} \ln 27}$$

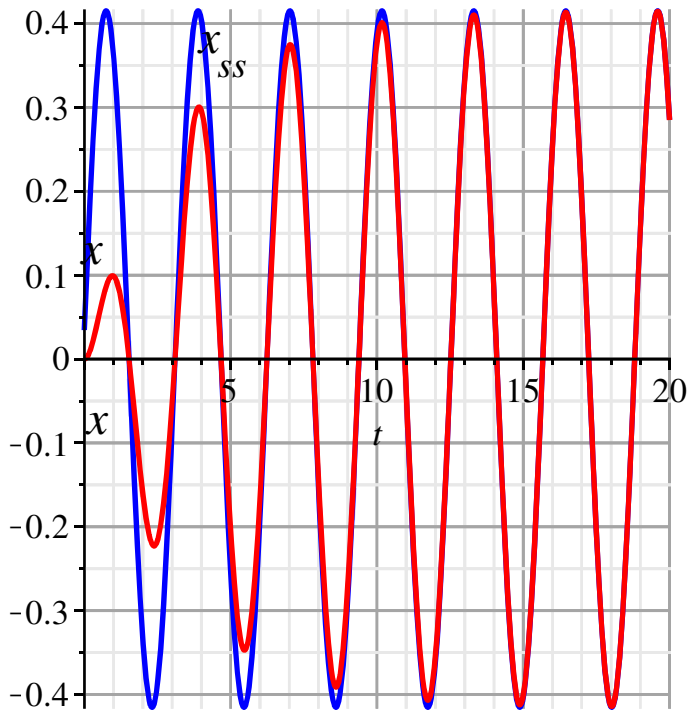
$$= 27^{\frac{1}{10}} + 3(27)^{-9/10} = 27^{\frac{1}{10}} \left( 1 + \frac{3}{(27)^{10/10}} \right)$$

$$= \frac{30}{27^{9/10}} = 10 \cdot \frac{3}{3^{9/10}} = 10 \cdot \frac{3 \cdot 3^{1/10}}{3^{9/10}} = \frac{10}{9} 3^{3/10}$$

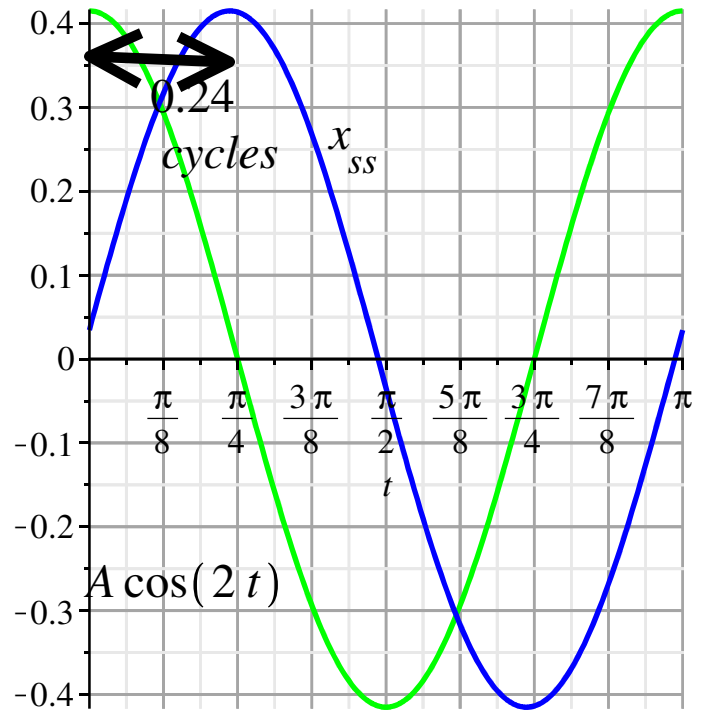
$$\approx 1.545$$

→ local min at  $\approx (0.33, 1.54)$  on graph

[MAT 2705 11F test 3 takehome plots: 1 c.

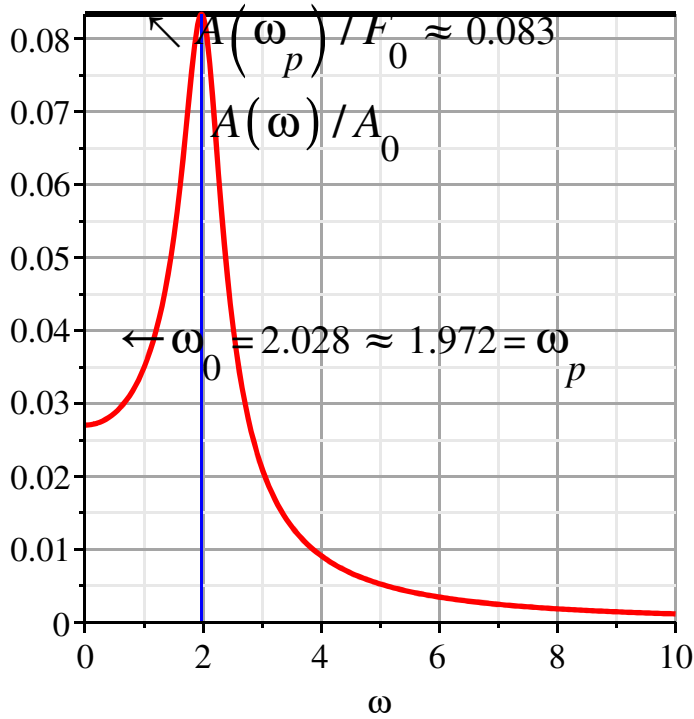


$\tau = 3, 5 \tau = 15$ , pixels merge by this time, amplitude  $\approx 0.415$  looks right



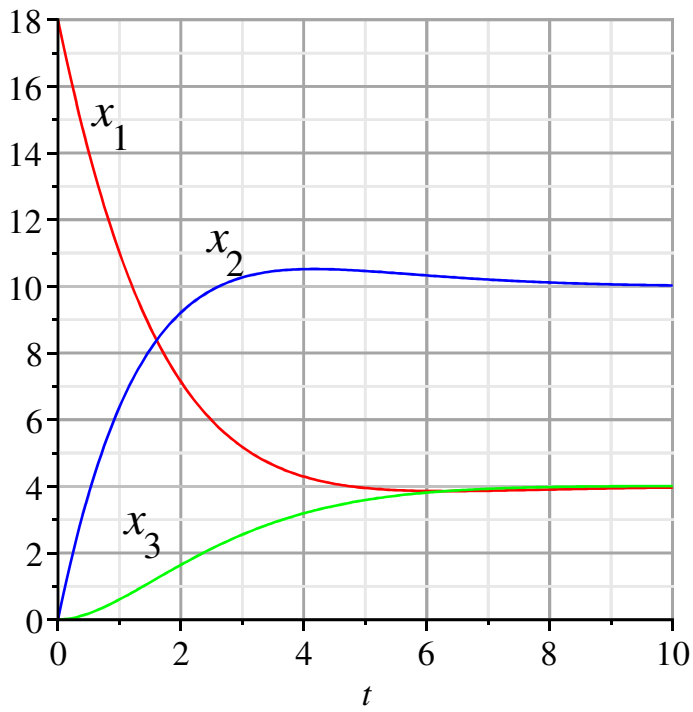
nearly 90 degrees out of phase (1/4 cycle), lagging behind (later in time), characteristic of resonance

[1f.

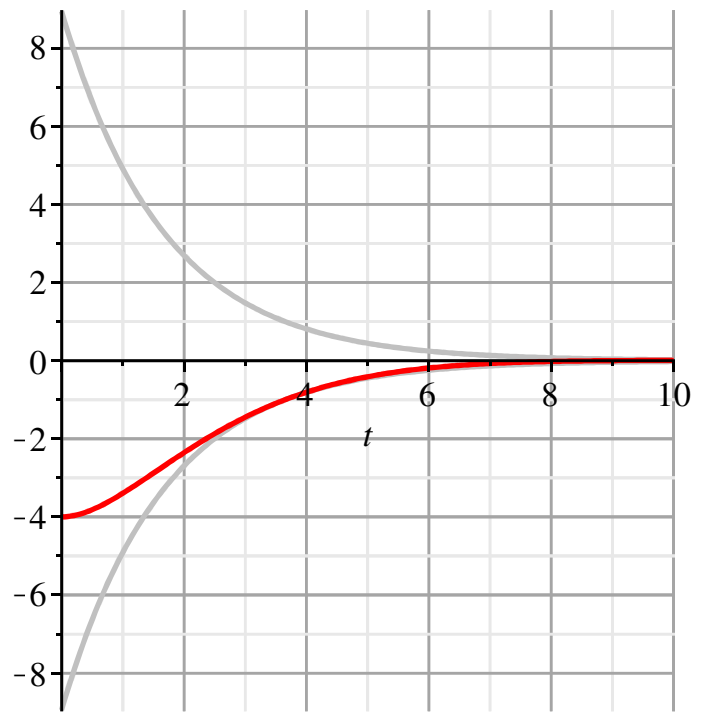


resonance peak in amplitude response function

[2e,f.

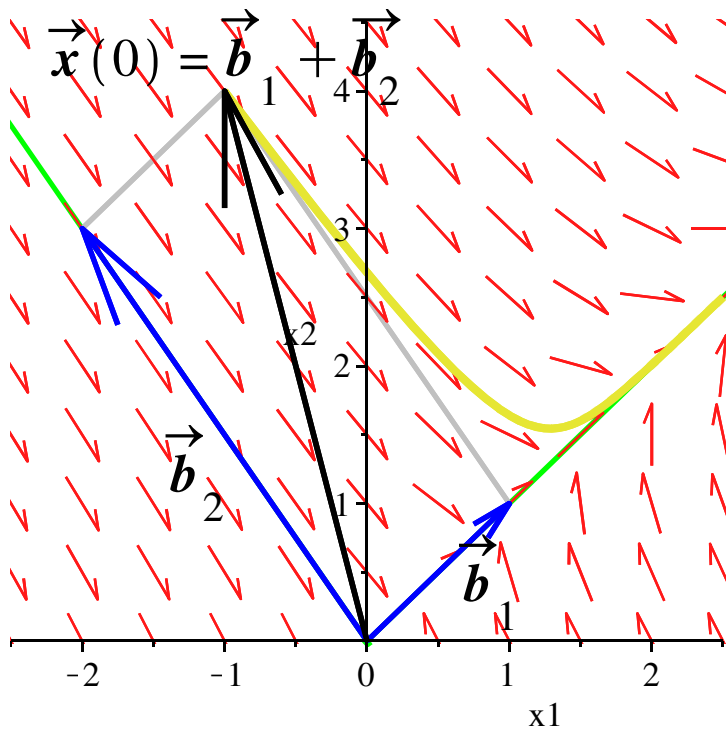


$\tau = 5/3, 5\tau \approx 8.33 \rightarrow 10$  for viewing window

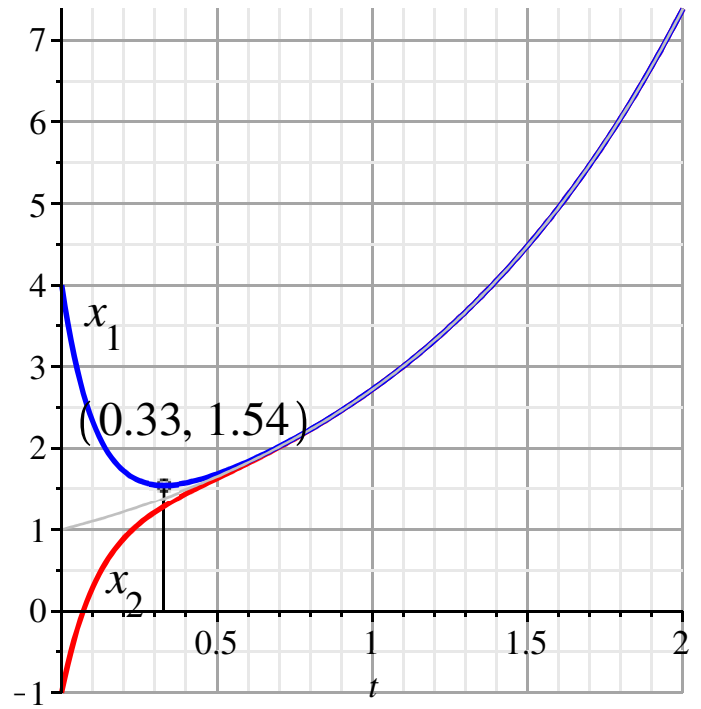


$x_{3\text{transient}}$  with exponential decaying envelope

[3c, d.



yellow solution curve approaches asymptote quickly, reaches the one percent level of  $y_2$  at about time 0.51



window appropriate for asymptote  $e^t$