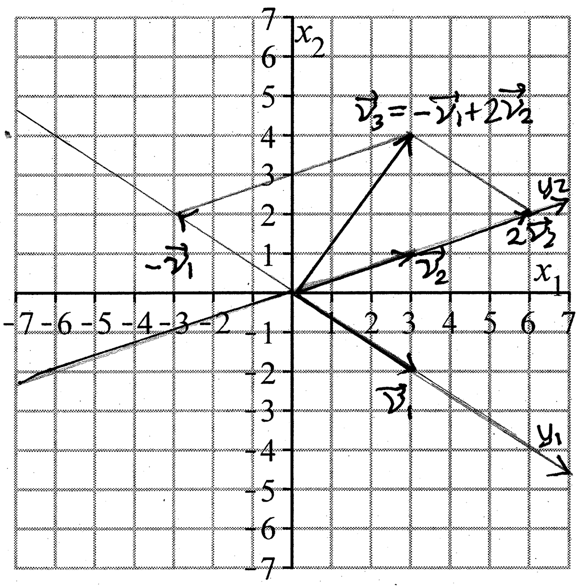
MAT2705 03/04 11F TEST 2 Answers

① a) $\langle x_1,x_2\rangle = \langle 3,4\rangle \to \langle y_1,y_2\rangle = \langle -1,2\rangle$

The properly labeled parallelogram below shows these to be the coefficients of $\vec v_1, \vec v_2$ which express $\vec v_3$.



b) $y_1\vec v_1 + y_2\vec v_2 = \vec v_3$

$$y_1\begin{bmatrix}3\\-2\end{bmatrix} + y_2\begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$$

$$\begin{bmatrix}3&3\\-2&+1\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$$

$$\begin{bmatrix}y_1\\y_2\end{bmatrix} = \begin{bmatrix}3&3\\-2&+1\end{bmatrix}^{-1}\begin{bmatrix}3\\4\end{bmatrix}$$

$$= \frac{1}{+3+6}\begin{bmatrix}1&-3\\2&3\end{bmatrix}\begin{bmatrix}3\\4\end{bmatrix} = \frac{1}{9}\begin{bmatrix}3-12\\6+12\end{bmatrix}$$

$$= \begin{bmatrix}-1\\2\end{bmatrix} \quad \text{so}$$

$$\vec v_3 = -\vec v_1 + 2\vec v_2$$

c) $\begin{bmatrix}3\\4\end{bmatrix} = -\begin{bmatrix}3\\-2\end{bmatrix} + 2\begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}-3+6\\2+2\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$ ✓

d) yes, they agree.

② a) $A = \begin{bmatrix}1&2&5&-1\\3&1&5&7\\2&1&4&4\\-1&0&-1&-3\end{bmatrix}$ $\vec x = \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$ $\vec b = \begin{bmatrix}8\\9\\7\\0\end{bmatrix}$

$$A\vec x = \vec b \to \langle A|\vec b\rangle = \begin{bmatrix}1&2&5&-1&8\\3&1&5&7&9\\2&1&4&4&7\\-1&0&-1&-3&0\end{bmatrix} \xrightarrow{\text{rred}}$$

$$\begin{bmatrix}1&0&1&3&0\\0&1&2&-2&0\\0&0&0&0&1\\0&0&0&0&0\end{bmatrix} \to 0 = 1 \text{ inconsistent system, no soln.}$$

② a) continued.

$\vec v_5$ cannot be expressed in terms of $\{\vec v_1, \vec v_2, \vec v_3, \vec v_4\}$ i.e., it does not lie in its span.

b) Let $\vec b = \langle 8,9,7,-2\rangle$: $A\vec x = \vec b$

$$\langle A|\vec b\rangle = \begin{bmatrix}1&2&5&-1&8\\3&1&5&7&9\\2&1&4&4&7\\-1&0&-1&-3&-2\end{bmatrix} \xrightarrow{\text{rred}}$$

$$\begin{array}{ccccc} L & L & F & F & \\ x_1 & x_2 & x_3 & x_4 & \end{array}$$
$$\begin{bmatrix}1&0&1&3&2\\0&1&2&-2&3\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$$

$x_1 + x_3 + 3x_4 = 2 \to x_1 = 2 - t_1 - 3t_2$

$x_2 + 2x_3 - 2x_4 = 3 \to x_2 = 3 - 2t_1 + 2t_2$

$0 = 0$

$0 = 0$

$x_3 = t_1,\ x_4 = t_2$

Therefore

$$\boxed{\langle 8,9,7,-2\rangle = (2-t_1-3t_2)\vec v_1 + (3-2t_1+2t_2)\vec v_2 + t_1\vec v_3 + t_2\vec v_4}$$

lies in the span of these 4 vectors.

c) $A\vec x = \vec 0$: → explicitly →

$$\begin{bmatrix}1&2&5&-1\\3&1&5&7\\2&1&4&4\\-1&0&-1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

matrix equation

$$\langle A|\vec 0\rangle = \begin{bmatrix}1&2&5&-1&0\\3&1&5&7&0\\2&1&4&4&0\\-1&0&-1&-3&0\end{bmatrix} \xrightarrow{\text{rred}}$$

$$\begin{array}{ccccc} L & L & F & F & \\ x_1 & x_2 & x_3 & x_4 & \end{array}$$
$$\begin{bmatrix}1&0&1&3&0\\0&1&2&-2&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix} \to$$

$x_1 + x_3 + 3x_4 = 0$ $\quad x_1 = -t_1 - 3t_2$

$x_2 + 2x_3 - 2x_4 = 0$ $\quad x_2 = -2t_1 + 2t_2$

$0 = 0$

$0 = 0$

$x_3 = t_1,\ x_4 = t_2$

$$\boxed{\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}-t_1-3t_2\\-2t_1+2t_2\\t_1\\t_2\end{bmatrix}} \text{ d) } = t_1\underbrace{\begin{bmatrix}-1\\-2\\1\\0\end{bmatrix}}_{\vec u_1} + t_2\underbrace{\begin{bmatrix}-3\\2\\0\\1\end{bmatrix}}_{\vec u_2}$$

$$\boxed{\{\vec u_1, \vec u_2\} = \{\langle -1,-2,1,0\rangle, \langle -3,2,0,1\rangle\}} = \text{basis soln space}$$

e) $\langle -1,-2,1,0\rangle$: $\boxed{-\vec v_1 - 2\vec v_2 + \vec v_3 = 0,}$

$\langle -3,2,0,1\rangle$: $\boxed{-3\vec v_1 + 2\vec v_2 + \vec v_4 = 0}$

f) only 2 of the 4 vectors form a lin. ind. set so their span is a $\boxed{\text{2-dimensional}}$ subspace of $\mathbb{R}^4$, i.e., a plane.

$\boxed{\{\vec v_1, \vec v_2\} = \{\langle 1,3,2,-1\rangle, \langle 2,1,1,0\rangle\}}$ is a basis of this plane although any 2 of the 4 vectors would do the job.