MAT2705-03/04 11F Test 1 Answers

① a) see diagram. my curve intersects the line $y=40$ three grid boxes left of the y axis, namely at $x \approx -0.6 (= 3\times 0.2)$.

b) $\frac{dy}{dx} = -ky + k10$

$e^{kx}\left[\frac{dy}{dx} + ky = 10k\right]$

$\left(e^{\int k\,dx} = e^{kx} \right.$

$\frac{d}{dx}(y e^{kx}) = 10k e^{kx}$

$y e^{kx} = \int 10k e^{kx}\,dx$

$= 10\cancel{k}\frac{e^{kx}}{\cancel{k}} + C$

$\boxed{y = e^{-kx}(10e^{kx} + C) = 10 + Ce^{-kx}}$ gen soln

c) $30 = y(0) = 10 + C \to C = 30-10 = 20$

$\boxed{y = 10 + 20e^{-kx}}$ IVP soln

d) $20 = y(1) = 10 + 20e^{-k}$

$10 = 20e^{-k}$

$\frac{1}{2} = e^{-k}$

$2 = e^{k}$

$\boxed{k = \ln 2 \approx 0.693}$

e) $y - 10 = 20e^{-kx}$

$\boxed{\tau = \frac{1}{k} = \frac{1}{\ln 2} \approx 1.44}$

f) $40 = y(t) = 10 + 20e^{-kx}$

$30 = 20e^{-kx}$

$\frac{3}{2} = e^{-kx}$

$\frac{2}{3} = e^{kx}$

$\ln\frac{2}{3} = kx$

$x = \frac{1}{k}\ln 2/3 = \tau \ln 2/3 = \boxed{\frac{\ln 2/3}{\ln 2}}$

$\approx -0.40547\,\tau$

$\boxed{\approx -0.585}$

g) $y - 10 = 20e^{-kx} = 0.2$

$0.01 = \frac{0.2}{20} = e^{-kx} \to e^{kx} = 100$

$kx = \ln .01$ $\quad x = \frac{1}{k}\ln 100 = \tau \ln 100 = \boxed{\frac{\ln 100}{\ln 2}} \approx 4.6\tau \approx \boxed{6.64}$

b) Maple produces exactly this form of the solution

② a) $\frac{dy}{dx} = \frac{x}{2\tan(y)}$ separable

$\int \tan y\,dy = \int \frac{x}{2}\,dx = \frac{x^2}{4} + C_1$

$-\ln|\cos y|$ exponentiate both sides after reversing sign

$e^{\ln|\cos y|} = e^{-\frac{x^2}{4} - C_1} = e^{-C_1}e^{-\frac{x^2}{4}}$

$|\cos y|$

$\cos y = \underbrace{(\pm e^{-C_1})}_{C} e^{-\frac{x^2}{4}} = Ce^{-\frac{x^2}{4}}$

$\boxed{y = \arccos(Ce^{-x^2/4})}$

$\frac{\pi}{3} = y(0) = \arccos(C) \to C = \cos\frac{\pi}{3} = \frac{1}{2}$

$\boxed{y = \arccos(\frac{1}{2}e^{-x^2/4})}$

c) Maple gives exactly this relation.

① a)