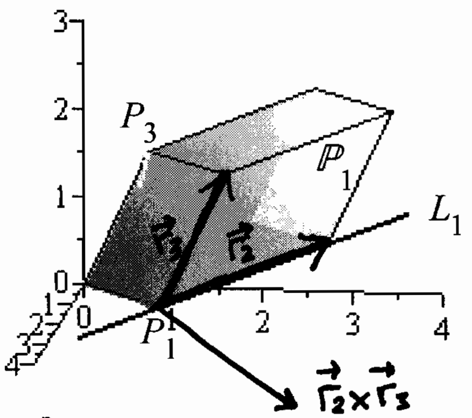
MAT2500-03/04 10S Quiz 2 Print Name (Last, First) ______________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P_1(1, 1, 0)$, $P_2(1, 2, 1)$, $P_3(2, 1, 2)$ and the parallelopiped formed from their three position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$.



a) Write the parametrized equations of the line L_1 through the point P_1 parallel to the position vector $\vec{r}_2$.

b) Find a normal vector $\vec{n}$ for the for the plane $\mathbb{P}_1$ which contains the right outside face of the parallelepiped shown in the figure.

c) Write the simplified equation for this plane.

d) Let $\vec{b} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3$ be the main diagonal of the parallepiped. Find the length b_{par} of its projection along the vector $\vec{r}_3$.

e) Evaluate $|\vec{r}_3 \times \vec{b}|$ and then $b_{perp} = |\vec{r}_3 \times \vec{b}|/|\vec{r}_3|$.

f) Does $b_{par}^2 + b_{perp}^2 = |\vec{b}|^2$ as it should?

▶ **solution**

Goes thru P_1:

a) $\vec r_0 = \langle 1,1,0\rangle$, $\vec a = \langle 1,2,1\rangle$ (direction of $\vec r_2$) $\rightarrow \vec r = \vec r_0 + t\vec a = \langle 1,1,0\rangle + t\langle 1,2,1\rangle = \langle 1+t, 1+2t, t\rangle$

$\boxed{\langle x,y,z\rangle = \langle 1+t, 1+2t, t\rangle}$ (or $x=1+t$, $y=1+2t$, $z=t$)

b) edge vectors $\vec r_2 = \langle 1,2,1\rangle$, $\vec r_3 = \langle 2,1,2\rangle$ ← (or use Maple)

$\vec r_1 \times \vec r_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \langle 2(2)-1(1), 1(2)-1(2), 1(1)-2(2)\rangle = \langle 3,0,-3\rangle$

$= 3\langle 1,0,-1\rangle$ pick $\boxed{\vec n = \langle 1,0,-1\rangle}$

c) $\vec r_0 = \langle 1,1,0\rangle$ goes thru P_1, orientation $\vec n$:

$0 = \vec n\cdot(\vec r - \vec r_0) = \langle 1,0,-1\rangle \cdot (\langle x,y,z\rangle - \langle 1,1,0\rangle) = \langle 1,0,-1\rangle\cdot\langle x-1,y-1,z\rangle$

$= (x-1) + (-1)(z) = x-z-1 \longrightarrow \boxed{x-z=1}$ (or $x-z-1=0$)

d) $\vec b = \langle 1,1,0\rangle + \langle 1,2,1\rangle + \langle 2,1,2\rangle = \langle 1+1+2, 1+2+1, 0+1+2\rangle = \langle 4,4,3\rangle$

$\vec r_3 = \langle 2,1,2\rangle$, $|\vec r_3| = \sqrt{4+1+4} = \sqrt 9 = 3$, $\hat r_3 = \frac13\langle 2,1,2\rangle$.

$b_{par} = |\hat r_3 \cdot \vec b| = |\frac13\langle 2,1,2\rangle\cdot\langle 4,4,3\rangle| = \frac13(8+4+6) = 6$

e) $\vec r_3 \times \vec b = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 4 & 4 & 3\end{vmatrix} = \langle 1(3)-(2)4, (2)4-2(3), 2(4)-1(4)\rangle = \langle -5, 2, 4\rangle$ (or use Maple)

$|\vec r_3| = \sqrt{4+1+4} = \sqrt 9 = 3$ $b_{perp} = \frac{|\langle -5,2,4\rangle|}{3} = \frac{\sqrt{25+4+16}}{3} = \frac{\sqrt{45}}{3} = \frac{3\sqrt5}{3} = \sqrt 5$

f) $|\vec b|^2 = 4^2+4^2+3^2 = 41$

$b_{par}^2 + b_{perp}^2 = 36 + 5 = 41$ ✓ yes!