MAT2500-03/04 10S Quiz 1 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P_1(1, 2)$, $P_2(-1, 4)$, $P_3(5, 3)$ in the plane:

a) On the reverse side of this sheet (left grid), draw in the three points and their position vectors, labeling each point and vector, and draw in the triangle the points determine, labeling the three sides by appropriate symbols for their difference vectors (put in arrow heads to indicate which direction your vector symbol for each side refers to).

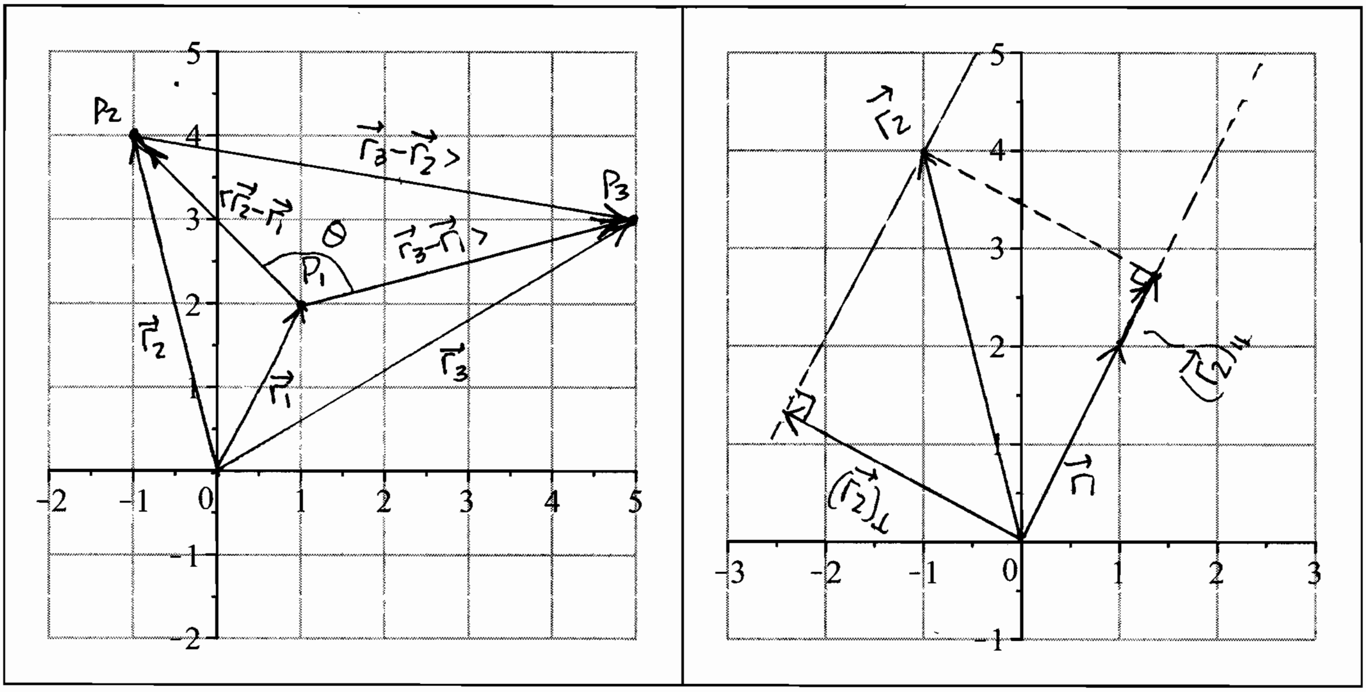
b) Evaluate the angle of the triangle at the vertex *P1* exactly (no decimals) and numerically to the nearest tenth of a degree. Does your result seem consistent with your graph? Explain.

c) On the reverse side of this sheet (right grid), draw in only the position vectors of the first two points and draw in the rectangle used to graphically project the second position vector parallel and perpendicular to the first vector, labeling the vectors and sides of the rectangle with appropriate notation for those projections. Estimate roughly the components of the two vector projections.

d) Now using appropriate notation, step by step (show every step starting from the initial vector components) evaluate the vector components of the parallel and perpendicular projections that you have drawn exactly and then to 2 decimal place accuracy.

e) How do your numerically evaluated exact vectors compare to your graphical estimates? Explain.

▶ solution



c)

a) $P_1(1,2)$, $P_2(-1,4)$, $P_3(5,3)$

b) $\overrightarrow{P_1P_3} = \vec r_3-\vec r_1 = \langle 5,3\rangle - \langle 1,2\rangle = \langle 4,1\rangle$ $|\overrightarrow{P_1P_3}| = \sqrt{4^2+1^2} = \sqrt{17}$

$\overrightarrow{P_1P_2} = \vec r_2-\vec r_1 = \langle -1,4\rangle - \langle 1,2\rangle = \langle -2,2\rangle = 2\langle -1,1\rangle$ $|\overrightarrow{P_1P_2}| = \sqrt{(-2)^2+2^2} = \sqrt{8} = 2\sqrt{2}$

$\widehat{P_1P_3} = \dfrac{\langle 4,1\rangle}{\sqrt{17}}$

$\widehat{P_1P_2} = \dfrac{\langle -1,1\rangle}{\sqrt{2}}$

$\widehat{P_1P_3}\cdot\widehat{P_2P_3} = \dfrac{\langle 4,1\rangle}{\sqrt{17}}\cdot\dfrac{\langle -1,1\rangle}{\sqrt{2}} = \dfrac{-4+1}{\sqrt{17}\sqrt{2}} = \dfrac{-3}{\sqrt{2}\sqrt{17}} = \cos\theta$

$\theta = \boxed{\arccos\left(\frac{-3}{\sqrt{2}\sqrt{17}}\right)} \approx 120.9637565 \approx \boxed{121.0^\circ}$

c) $(\vec r_2)_\parallel \approx \langle 1.35, 2.75\rangle$

$(\vec r_2)_\perp \approx \langle -2.4, 1.3\rangle$

e) pretty close! good enough.

d) $\vec r_1 = \langle 1,2\rangle$ $\hat r_1 = \dfrac{\langle 1,2\rangle}{\sqrt{5}}$

$\vec r_2 = \langle -1,4\rangle$

$\vec r_2\cdot\hat r_1 = \langle -1,4\rangle\cdot\dfrac{\langle 1,2\rangle}{\sqrt5} = \dfrac{-1+8}{\sqrt5} = \dfrac{7}{\sqrt5}$ scalar projection along $\vec r_1$

$(\vec r_2)_\parallel = (\vec r_2\cdot\hat r_1)\,\hat r_1 = \dfrac{7}{\sqrt5}\,\dfrac{\langle 1,2\rangle}{\sqrt5} = \dfrac{7}{5}\langle 1,2\rangle = \boxed{\left\langle \frac{7}{5},\frac{14}{5}\right\rangle \approx \langle 1.4, 2.8\rangle}$ vector projection along $\vec r_1$

$(\vec r_2)_\perp = \vec r_2 - (\vec r_2)_\parallel = \langle -1,4\rangle - \left\langle \frac75,\frac{14}{5}\right\rangle = \left\langle \frac{-5-7}{5}, \frac{20-14}{5}\right\rangle$

$= \boxed{\left\langle \frac{-12}{5},\frac{6}{5}\right\rangle \approx \langle -2.4, 1.2\rangle}$ vector projection perpendicular to $\vec r_1$

NOTATION

either $\overrightarrow{P_1P_2}$ or $\vec r_2-\vec r_1$ type notation works to identify vectors. use 1 or the other! same with $\vec a_\parallel, \vec b_\perp$ or $\vec b_{par}, \vec b_{perp}$ — use notation that identifies vectors in context of problem