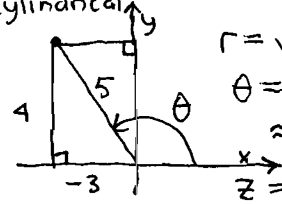
MAT2500-03/04 10S Final Exam Answers(1)

① $(x,y,z) = (-3,4,12)$

a) cylindrical



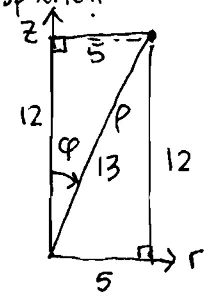
$r=\sqrt{(-3)^2+4^2}=\sqrt{25}=5$

$\theta = \pi - \arctan\left(\frac{4}{3}\right)$

$\approx 126.9°$

$z=12$

b) spherical



$\rho=\sqrt{(-3)^2+4^2+12^2}=\sqrt{5^2+12^2}=13$

$\varphi = \arctan\frac{5}{12} = \arccos\frac{12}{13} = \arcsin\frac{5}{13}$

$\approx 22.6°$

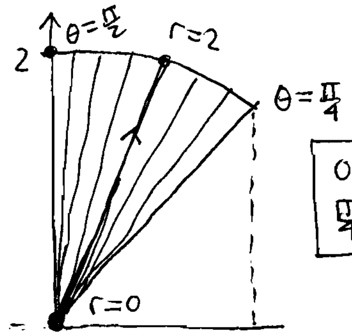
θ same as above

Both angles are believable

θ should be a bit less that $90°+45°=135°$

φ should be somewhatless than $30°$

② a)



$$\boxed{\begin{array}{c} 0\le r\le 2 \\ \frac{\pi}{4}\le\theta\le\frac{\pi}{2}\end{array}}$$

b) $A=\int_{\pi/4}^{\pi/2}\int_0^2 1\cdot r\,dr\,d\theta = \int_0^2 r\,dr\int_{\pi/4}^{\pi/2} 1\,d\theta$

$= \left.\frac{r^2}{2}\right|_0^2 \left.\theta\right|_{\pi/4}^{\pi/2} = 2\left(\frac{\pi}{4}\right) = \boxed{\frac{\pi}{2}}$

$A_x = \int_{\pi/4}^{\pi/2}\int_0^2 (r\cos\theta)\, r\,dr\,d\theta = \int_0^2 r^2dr\int_{\pi/4}^{\pi/2}\cos\theta\,d\theta$

$= \left.\frac{r^3}{3}\right|_0^2 \left.\sin\theta\right|_{\pi/4}^{\pi/2} = \boxed{\frac{8}{3}\left(1-\frac{1}{\sqrt2}\right)}$

$A_y = \int_{\pi/4}^{\pi/2}\int_0^2 (r\sin\theta)\, r\,dr\,d\theta = \int_0^2 r^2dr\int_{\pi/4}^{\pi/2}\sin\theta\,d\theta$

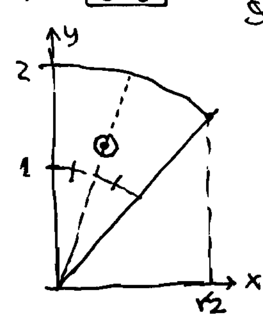
$= \left.\frac{r^3}{3}\right|_0^2 \left.(-\cos\theta)\right|_{\pi/4}^{\pi/2} = \frac{8}{3}\left(0+\frac{1}{\sqrt2}\right) = \boxed{\frac{8}{3\sqrt2}}$

$(\bar{x},\bar{y}) = \left(\frac{8}{3}\left(1-\frac{1}{\sqrt2}\right)\frac{2}{\pi},\ \frac{8}{3\sqrt2}\left(\frac{2}{\pi}\right)\right)$

$= \left(\frac{16}{3\pi}\left(1-\frac{1}{\sqrt2}\right),\ \frac{16}{3\pi\sqrt2}\right)$

$\approx (0.497, 1.200)$

see diagram →



on anglebisector
past midpoint in radial direction
(more area at higher radii)

③ a) $\vec{F} = \langle F_1, F_2\rangle = \langle 2x-2y, -2x+2y\rangle = \vec\nabla f$

$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = \frac{\partial}{\partial x}(-2x+2y) - \frac{\partial}{\partial y}(2x-2y) = -2+2$
$= 0$ ✓

b) $\int\left[\frac{\partial f}{\partial x} = 2x-2y\right]dx \to f = \int 2x-2y\,dx$
$= x^2-2xy+C(y)$

$\frac{\partial f}{\partial y} = -2x+2y$ $\qquad \frac{\partial f}{\partial y} = -2x+C'(y)$

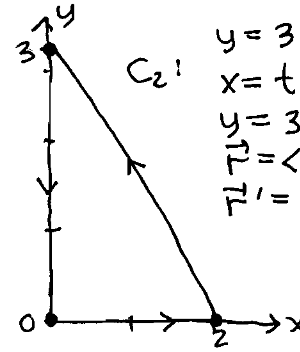
$-2x+C'(y) = -2x+2y \to \int[C'(y)=2y]\,dy$

$C(y) = \int 2y\,dy = y^2+k$ (set $k=0$)

$\boxed{f(x,y) = x^2-2xy+y^2}$

c) $\int_C \vec{F}\cdot d\vec{r} = f(0,3)-f(2,0)$
$= 3^2-4^2 = 9-4 = \boxed{5}$

d)



C_2: $y=3-\frac{3x}{2}$
$x=t=2..0$
$y=3-3t/2$
$\vec{r}=\langle t, 3-\frac{3t}{2}\rangle$
$\vec{r}\,'=\langle 1,-\frac{3}{2}\rangle$

C_3: $x=0$
$y=t=3..0$
$\vec{r}=\langle 0,t\rangle$
$\vec{r}\,'=\langle 0,1\rangle$

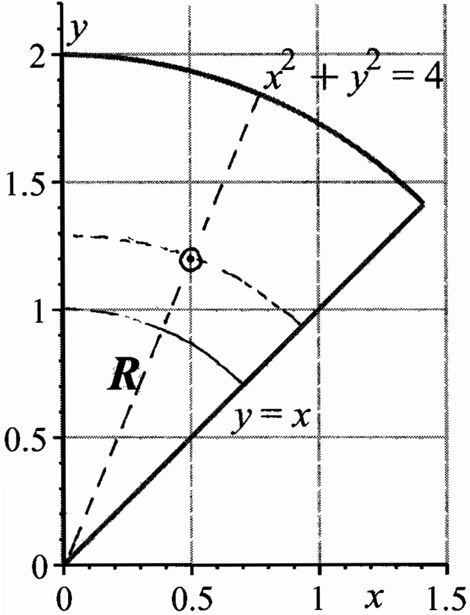
C_1: $x=t=0..2$
$y=0$
$\vec{r}=\langle t,0\rangle$, $\vec{r}\,'=\langle 1,0\rangle$

C_1: $\vec{F}(\vec{r}(t)) = \langle 2t-0, -2t+0\rangle = 2\langle t,-t\rangle$
$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = 2t$
$\int_{C_1}\vec{F}\cdot d\vec{r} = \int_0^2 2t\,dt = \left.t^2\right|_0^2 = \boxed{4}$

C_3: $\vec{F}(\vec{r}(t)) = \langle 0-2t, 0+2t\rangle = \langle -2t, 2t\rangle$
$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = 2t$
$\int_{C_3}\vec{F}\cdot d\vec{r} = \int_3^0 2t\,dt = \left.t^2\right|_3^0 = \boxed{-9}$

C_2: $\vec{F}(\vec{r}(t)) = \langle 2t-2(3-\frac{3t}{2}), -2t+2(3-\frac{3t}{2})\rangle$
$= \langle 5t-6, 6-5t\rangle$
$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = (5t-6)-\frac{3}{2}(6-5t) = \frac{5}{2}(5t-6)$
$\int_{C_2}\vec{F}\cdot d\vec{r} = \int_2^0 \frac{5}{2}(5t-6)\,dt = \frac{5}{2}\left(\frac{5t^2}{2}-6t\right)\Big|_2^0$
$= -\frac{5}{2}(5\cdot2-12) = \boxed{+5}$ [agrees part c)]

$\oint_C \vec{F}\cdot d\vec{r} = 4+5-9 = \boxed{0}$ ✓



y
2
1.5
1
0.5
0
0
0.5
1
x
1.5
$x^2 + y^2 = 4$
R
$y = x$