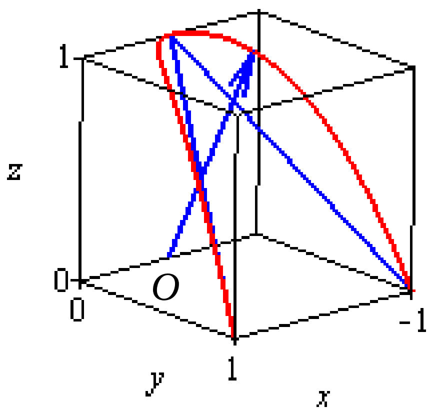
MAT2500-03/04 10S Test 1  Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad). You are encouraged to use technology to check all of your hand results.




The parametrized curve $\vec{r}(t) = \langle t, t^2, 1 - t^4 \rangle$, $-1 \le t \le 1$ is shown in the figure together with the vector $\vec{r}(-1/2)$ and line segments connecting the 3 points $\vec{r}(-1)$, $\vec{r}(0)$, $\vec{r}(1)$, showing that this is not a plane curve.

a) Evaluate and simplify $\vec{r}\,'(t)$, $\vec{r}\,''(t)$, $|\vec{r}\,'(t)|$, $\vec{T}(t)$, $|\vec{r}\,''(t)|$ and their values (including $\vec{r}(t)$) at $t = -1/2$.

b) Evaluate the unit vector $\vec{B}(t)$ in the direction of $\vec{r}\,'(t) \times \vec{r}\,''(t)$.

c) Write the equation of the plane through $\vec{r}(-1/2)$ containing the tangent vector and the second derivative there.

d) Write the parametrized equations of the tangent line through $\vec{r}(-1/2)$.

e) Find the coordinates of the point on the top of the box in the figure where that tangent line passes through this plane (namely the plane $z = 1$).

f) Write down an integral formula for the length of the curve $\vec{r}(t)$ from $t = 0$ to $t = 1$. Numerically evaluate this using technology. and compare your result to the length of the straight line segment between its endpoints (which is the shortest distance between those points!). How close are the latter numbers, i.e., what is the ratio of the length of the curve to the length of the straight line? What percent longer is the curve? Does this seem reasonable?

g) Evaluate the scalar tangential projection $a_T(-1/2)$ along $\vec{T}(-1/2)$ of the acceleration $\vec{a}(-1/2) = \vec{r}\,''(-1/2)$ and its scalar normal projection $a_N(1) = |\vec{T}(-1/2) \times \vec{a}(-1/2)|$ exactly. Does the sum of their squares equal the square of the magnitude of the acceleration as it should? Is that sum a lucky number?

## ▶ solution

## ▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:                                        Date: